(a) From Eq. (14.25) of the previous problem, we have that in the frame in which the proton is at rest:

$$q^\mu = (k^\mu - k'^\mu) = \left(\frac{(E-E')}{c}, (\vec{k} - \vec{k}')\right),$$
$$P^\mu = (m_p c, 0), \tag{14.42}$$

(Q^μ in the problem should be identified with q^μ. The four-vectors used in this specific problem are defined with factors of c different from what is given in the Appendix.) Through direct evaluation, we have

$$P \cdot q c^2 = m_p c \times \frac{(E-E')}{c} \times c^2 = m_p \nu c^2. \tag{14.43}$$

This is a Lorentz invariant quantity, and is therefore independent of the frame of reference. In particular, it holds even in the frame in which the proton has an exceedingly large spatial momentum (the "infinite momentum frame").

(b) Let us assume that only one parton of the proton participates in the reaction, and that its four-momentum is given by (see Fig. 14.17):

$$P^\mu_{\text{parton}} = xP^\mu, \tag{14.44}$$

where x represents the fraction of the proton's four-momentum that is carried by the parton. The effective reaction (see accompanying Fig. 14.17) is given by

$$e^- + \text{parton} \to e^- + \text{parton}. \tag{14.45}$$

Denoting by $P^\mu_{\text{parton}}, P'^\mu_{\text{parton}}$ the four-momenta of the incident and the final-state partons, respectively, conservation of energy-momentum

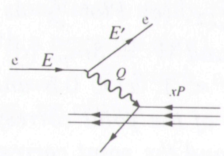

Fig. 14.17. Deep-inelastic scattering (DIS) in the infinite momentum frame.

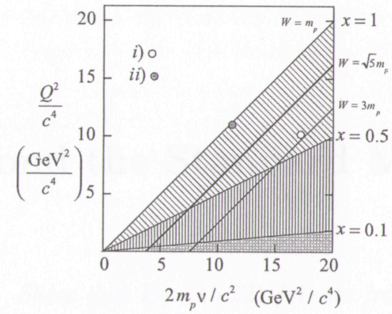

Fig. 14.18. Kinematics of deep-inelastic scattering.

leads to

$$P^\mu_{\text{parton}} + k^\mu = P'^\mu_{\text{parton}} + k'^\mu$$
$$\text{or} \quad (xP^\mu + (k^\mu - k'^\mu)) = P'^\mu_{\text{parton}}, \tag{14.46}$$

where we have used Eq. (14.44). If we assume the parton to be massless, then squaring the above relationship we obtain

$$(xP + (k - k'))^2 = P'^2 \approx 0$$
$$\text{or} \quad (xP + q)^2 = x^2P^2 + q^2 + 2xP \cdot q \approx 0 \tag{14.47}$$
$$\text{and} \quad x^2 m_p^2 c^2 - \frac{Q^2}{c^2} + 2x m_p \nu \approx 0,$$

where we have used Eq. (14.43) as well as the definition in (14.29). If we assume that

$$Q^2 \gg x^2 m_p^2 c^4, \tag{14.48}$$

then, we can neglect the first term in the above equation and this leads to the expected result

$$x = \frac{Q^2}{2m_p \nu c^2}. \tag{14.49}$$

(c) The plots of interest are given in Fig. 14.18.