8. Accelerators

Problem 8.1 *Protons are accelerated in a cyclotron by an electric field with oscillating frequency of* 8 MHz. *If the diameter of the magnet is* 1 m, *calculate the value of magnetic field and the maximum energy that the protons can reach.*

Protons are accelerated in a cyclotron of radius 0.5 m, and an 8 MHz acceleration frequency ($\omega = 2\pi f = 5 \times 10^7$ rad/s). Protons will circulate at an 8 MHz frequency for a magnetic field of:

$$B = \frac{m\omega}{Q} = \frac{(1.67 \times 10^{-27}\ \text{kg})(5.0 \times 19^7\ \text{rad/s})}{1.6 \times 10^{-19}\text{C}} = 0.52\ \text{T} \qquad (8.1)$$

or, using eV units for the mass

$$B = \frac{mc^2\omega}{Qc^2} = \left(\frac{mc^2}{Q}\right)\frac{\omega}{c^2}$$

$$= (940 \times 10^6\,\text{eV})\frac{(5.0 \times 10^7\,\text{rad/s})}{9 \times 10^{16}\,\text{m}^2/\text{s}^2} = 0.52\,\text{T}. \qquad (8.2)$$

The maximum kinetic energy

$$T = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{\omega R}{c}\right)^2$$

$$= 0.5(940\,\text{MeV})\left[\frac{2.5 \times 10^7}{3 \times 10^8}\right]^2 = 3.3\,\text{MeV}. \qquad (8.3)$$

Problem 8.2 *To achieve an energy of* 20 TeV, *each of the SSC main rings was to contain about* 4000 *dipole magnets, each* 16*-meters long, with a field of* 7 T. *This means that over half of the* $\approx$60 *mile SSC tunnel was to be taken up by dipoles. If you were to build a single synchrotron for use in fixed-target collisions of equivalent energy in the center of mass* ($\sqrt{s} = 40$ TeV), *and used a similar magnet design, how long would your tunnel have to be?*

The SSC was designed to have center mass energy of 40 TeV. A fixed-target machine with a beam energy E' has a center-of-mass energy

$$\sqrt{s} = \sqrt{2mE'} \qquad (8.4)$$

to have the same center-of-mass energy as the SSC, the energy of a fixed target beam must be:

$$E' = \frac{(\sqrt{2})^2}{2m_pc^2} = \frac{1600 \times 10^6\,\text{GeV}^2}{2\,\text{GeV}} = 9 \times 10^5\,\text{TeV}. \qquad (8.5)$$

The circumference of the circular tunnel will scale with the energy

$$C' = C\frac{E'}{E} = (60\,\text{miles})\frac{8 \times 10^5\,\text{TeV}}{20\,\text{TeV}} = 2.4 \times 10^6\,\text{miles}. \qquad (8.6)$$