CHAPTER 9
Elementary Particles

Lecture Notes For
PHYS 415
Introduction to Nuclear and Particle Physics

To Accompany the Text
Introduction to Nuclear and Particle Physics, 2nd Ed.
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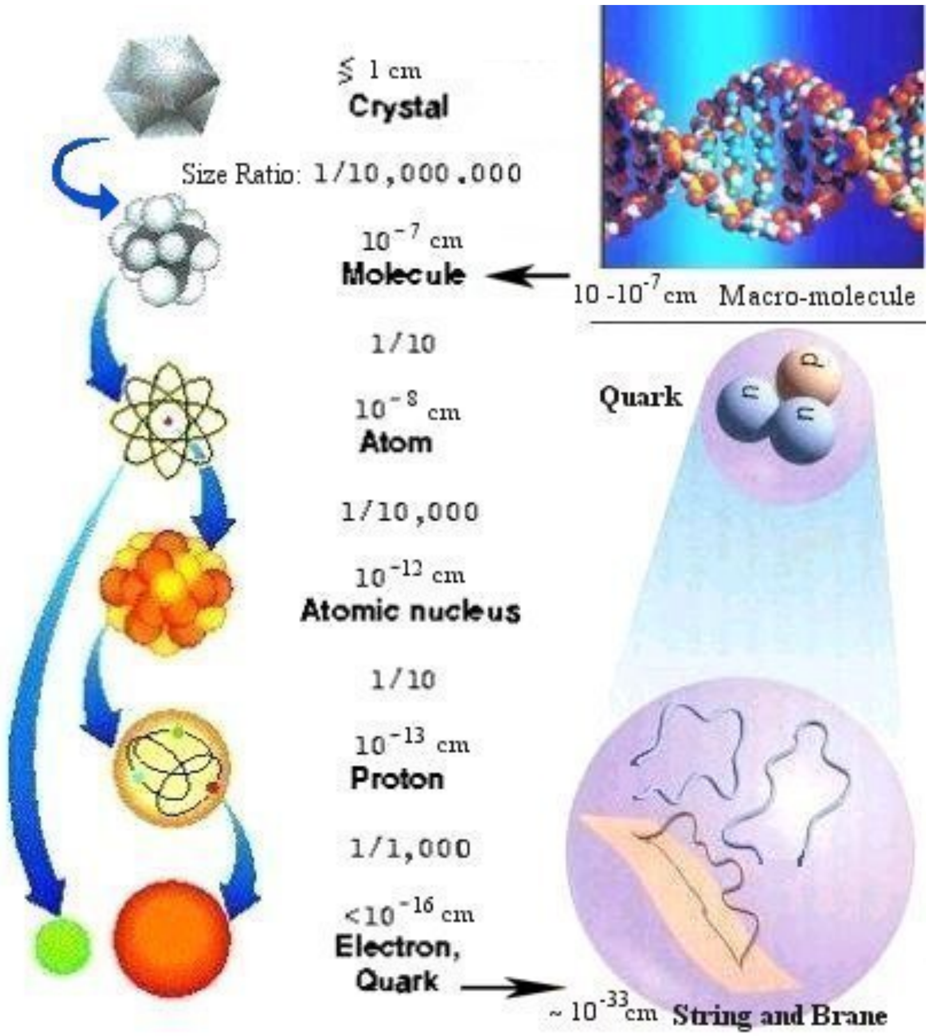
World Scientific

Introduction

- How do we know when we have uncovered the true fundamental constituents of matter?
  - Atoms were once thought to be fundamental.
  - Atoms were then thought to consist of nuclei (protons and neutrons) and electrons.
  - We now know that protons and neutrons are not fundamental, but are made of quarks.
  - Are quarks and electrons made of yet other stuff?
- Our definition of fundamental is relative to the distance scale we are able to probe.
- Will future experiments reveal substructure to quarks and electrons?

Distance Scales



From http://universe-review.ca/I15-01-domain.jpg

Small Distance $\Rightarrow$ High Energy

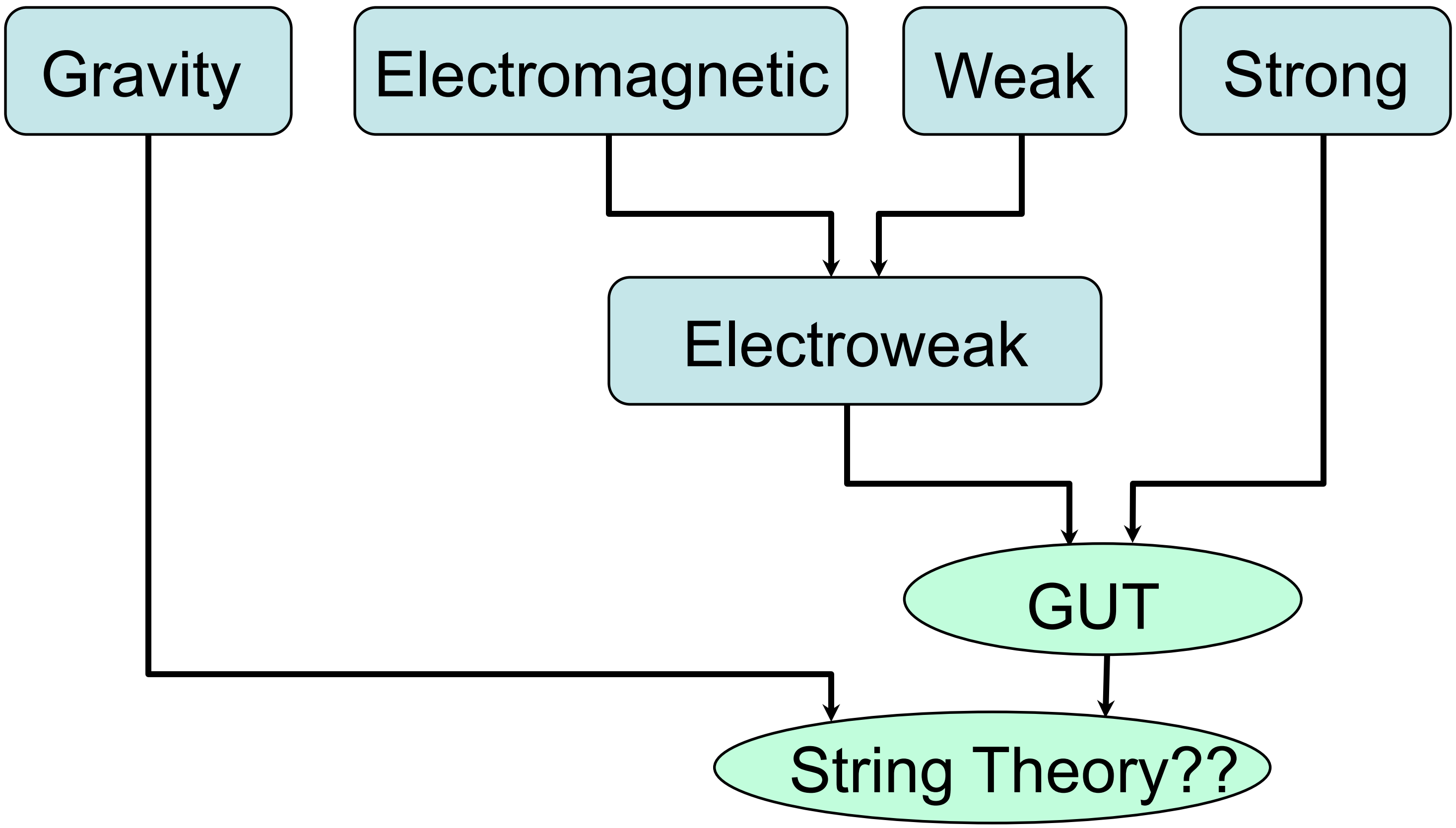
- To probe distance Δr requires transverse momentum transfer Δp_T:

$$\Delta r \approx \frac{\hbar}{\Delta p_T} = \frac{\frac{\hbar c}{c}}{\Delta p_T} = \frac{0.197\ \frac{\text{GeV}}{\text{c}}\text{ - fm}}{\Delta p_T} \approx \left(\frac{0.2\ \frac{\text{GeV}}{\text{c}}}{\Delta p_T} \right) \text{fm}$$

- So with a momentum transfer of 2 GeV/c, we can probe distances of order 0.1 fm.

Forces

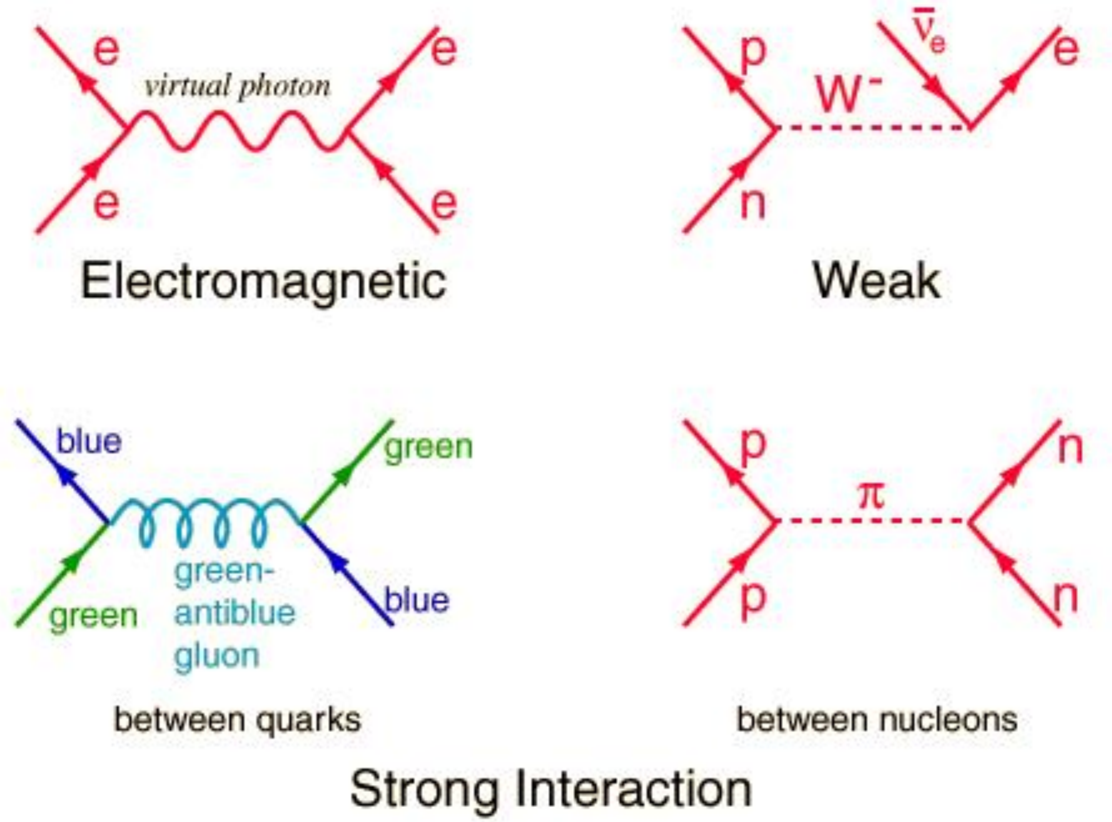
- At present, we have four fundamental forces:



Force Carriers

- In a quantum theory, each force is mediated by the exchange of some particle (**gauge boson**).
  - Electromagnetic: Photon
  - Weak: W^+, W^-, Z^0
  - Strong: gluon
  - Gravity: graviton (?)
- The ranges (and strengths) of the forces vary:
  - Gravity and electromagnetic: infinite range $\Rightarrow$ massless carrier.
  - Weak: short range $\Rightarrow$ massive carrier.
  - Strong: short range. As far as we know the gluon is massless as predicted by quantum field theory (gauge invariance). The range of the force is instead related to **confinement**: *gluons interact with themselves*. This also implies that gluons are confined to hadrons, leaving mesons as the effective carriers of the nuclear force.

Interactions



From http://universe-review.ca/I15-53-electroweak.jpg

Strengths of the Forces

- To compare the force strengths, we can use non-relativistic potentials as a guide.
- Consider two protons, separated by a distance *r* :

$$V_{\text{em}}(r) = \frac{e^2}{r}$$

$$V_{\text{grav}}(r) = \frac{G_N m^2}{r}$$

- In momentum-space (i.e. Fourier Transform):

$$V_{\text{em}}(q) = \frac{e^2}{q^2}$$

$$V_{\text{grav}}(q) = \frac{G_N m^2}{q^2}$$

Gravity vs. Electromagnetic Force

- Taking the ratio:

$$\frac{V_{\text{em}}}{V_{\text{grav}}} = \frac{e^2}{G_N m^2} = \left(\frac{e^2}{\hbar c}\right)\frac{1}{\left(mc^2\right)^2}\frac{\hbar c \times c^4}{G_N}$$

$$\approx \left(\frac{1}{137}\right)\frac{1}{(1\ \text{GeV})^2}\frac{10^{39}\ \text{GeV}^2}{6.7} \approx 10^{36}$$

- So for the "elementary" charged particles, gravity is far weaker than the electromagnetic force.

Weak and Strong Forces

- For the weak and strong forces, consider the Yukawa potential:

$$V_{\text{strong}} = \frac{g_s^2}{r} e^{-\frac{m_\pi c^2 r}{\hbar c}}$$

$$V_{\text{wk}} = \frac{g_{wk}^2}{r} e^{-\frac{m_W c^2 r}{\hbar c}}$$

- In momentum space:

$$V_{\text{strong}} = \frac{g_s^2}{q^2 + m_\pi^2 c^2}$$

$$V_{\text{wk}} = \frac{g_{wk}^2}{q^2 + m_W^2 c^2}$$

Weak and Strong Coupling Constants

- From experiment: $\dfrac{g_s^2}{\hbar c} \approx 15, \qquad \dfrac{g_{\mathrm{wk}}^2}{\hbar c} \approx 0.004$

- Also: $m_\pi \approx 0.140\ \mathrm{GeV}/c^2, \qquad m_W \approx 80\ \mathrm{GeV}/c^2$

- However, there is a momentum scale which does not cancel in taking ratios. Choose the proton mass to define the scale:

$$q^2c^2 = \left(mc^2\right)^2 = \left(1\ \mathrm{GeV}\right)^2$$

Strengths Compared

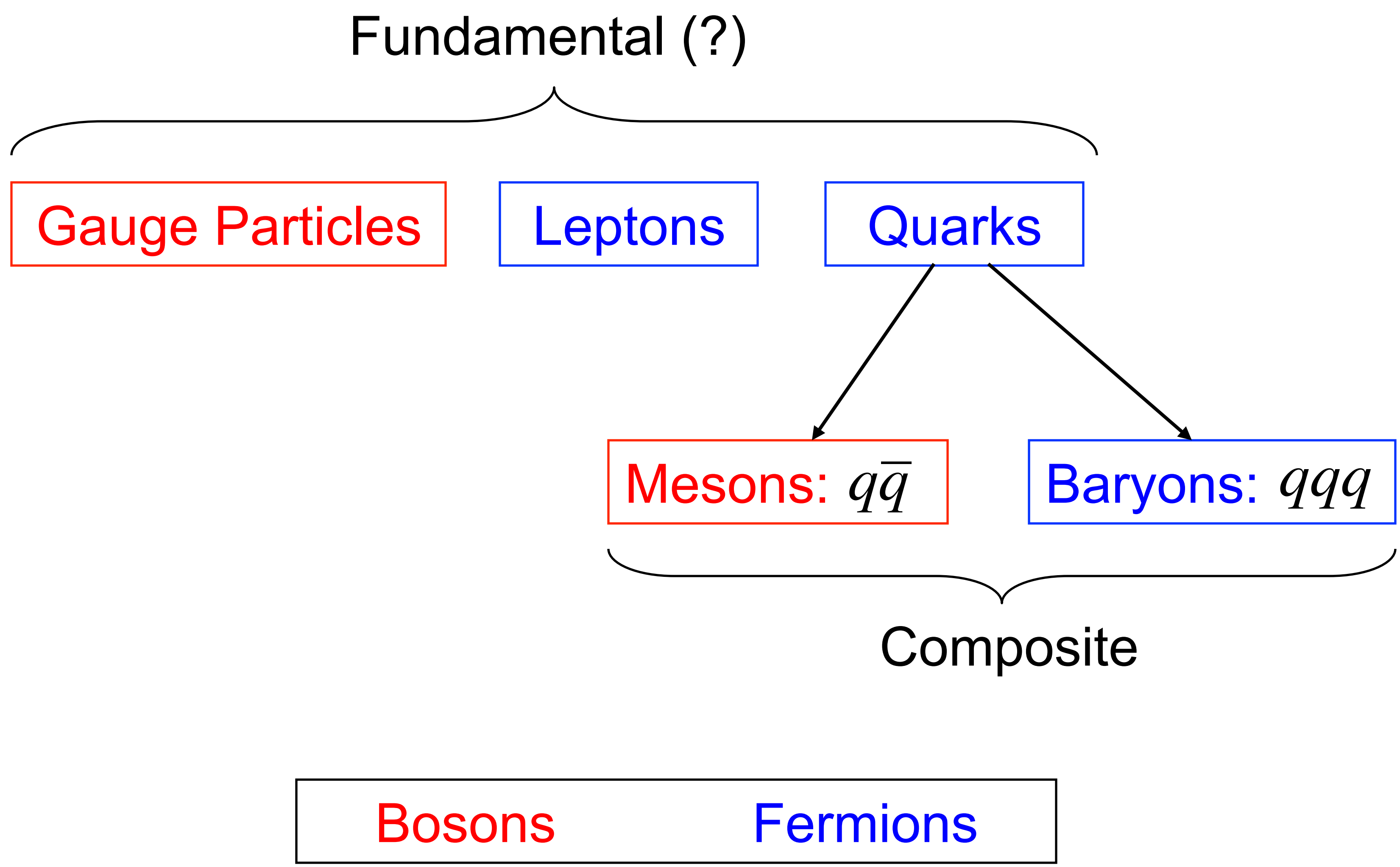
- Taking ratios:

$$\frac{V_{\text{strong}}}{V_{\text{em}}} = \frac{g_s^2}{\hbar c}\frac{\hbar c}{e^2}\frac{q^2}{q^2 + m_\pi^2 c^2} = \frac{g_s^2}{\hbar c}\frac{\hbar c}{e^2}\frac{m^2 c^4}{m^2 c^4 + m_\pi^2 c^4} \approx 15 \times 137 \times 1 \approx 2 \times 10^3$$

$$\frac{V_{\text{em}}}{V_{\text{wk}}} = \frac{e^2}{\hbar c}\frac{\hbar c}{g_{\text{wk}}^2}\frac{m^2 c^4 + m_W^2 c^4}{m^2 c^4} \approx \frac{1}{137}\frac{1}{0.004}(80)^2 \approx 1.2 \times 10^4$$

- For momentum scales ~ $m_W c$, the weak and electromagnetic forces become comparable. This suggests the possibility of **unification**.
- The differing strengths of forces also implies different time scales for various interactions: ~10^{-24} s for strong, ~10^{-20}-10^{-16} s for EM and 10^{-13}-10^{-6} for weak.

Particle Classifications



Leptons

- e^-, μ^-, τ^- along with their associated neutrinos and antiparticles/antineutrinos
- Fundamental (as far as we know)
- Masses: ~ 3 eV/c^2 (ν) up to 1.78 GeV/c^2 (τ)
- Fermions: half-integral spin
- Participate in
  - Gravity
  - Electromagnetism (except neutrinos/antineutrinos)
  - Weak Interaction

Mesons

- π^+ π^-, π^0, K^+, K^-, K^0, ρ^+, ρ^-, ρ^0, …
- Not fundamental: composed of quark-antiquark pair
- Masses: 135 MeV/c^2 up to few GeV/c^2
- Bosons: integral spin
- Participate in
  - Gravity
  - Electromagnetism (even neutral mesons, since quarks are charged)
  - Weak Interaction
  - Strong Interaction

Baryons

- p, n, Λ^0, Σ^+, Σ^-, Σ^0, Δ^{++}, Δ^0, …
- Not fundamental: composed of three quarks
- Masses: 938 MeV/c^2 up to few GeV/c^2
- Fermions: half-integral spin
- Participate in
  - Gravity
  - Electromagnetism (even neutral baryons, since quarks are charged)
  - Weak Interaction
  - Strong Interaction

Bosons and Fermions

- Bosons
  - Have integral spin
  - Wave function of a system of identical bosons is symmetric under interchange of any two particles:

$$\Psi_B(x_1, x_2, x_3, \ldots, x_n) = \Psi_B(x_2, x_1, x_3, \ldots, x_n) = \cdots$$

- Fermions
  - Have half-integral spin
  - Wave function of a system of identical fermions is anti-symmetric under interchange of any two particles:

$$\Psi_F(x_1, x_2, x_3, \ldots, x_n) = -\Psi_F(x_2, x_1, x_3, \ldots, x_n) = \cdots$$

  - Therefore they satisfy the Pauli Exclusion Principle: No two identical fermions can occupy the same quantum state.

Antiparticles

- Antiparticles have same mass and spin, but otherwise opposite quantum numbers, of corresponding particles.
  - Example:
    - Proton (p): Spin = 1/2, Charge = +e, Baryon # = +1
    - Antiproton $\left(\overline{p}\right)$: Spin = 1/2, Charge = -$e$, Baryon # = -1
  - Example:
    - Neutron (n): Spin = 1/2, Charge = 0, Baryon # = +1
    - Antineutron $\left(\overline{n}\right)$: Spin = 1/2, Charge = 0, Baryon # = -1
  - Example:
    - Neutral pion (π^0): Spin = 0, Charge = 0, …
    - Antiparticle: Same as π^0
  - Example: neutrino. Is it distinct from its antiparticle??

Quantum Numbers

- Quantum numbers are associated with conserved quantities.
- Certain quantum numbers seem to be universally conserved whereas others are conserved only in certain interactions.
- Always conserved (as far as we know):
  - Electric charge
  - Baryon #
  - Lepton # …
- Conserved only in certain interactions:
  - Isospin: conserved in strong, but not EM & weak, interactions.
  - Strangeness: conserved in strong and EM, but not weak, interactions.

Baryon Number

- Though the following reaction does not violate energy, charge or angular momentum conservation, it does not appear to occur (probability < 10^{-40} /sec):

$$p \not\to e^{+} + \pi^{0}$$

- Define **baryon number**:
  - +1 for all baryons (protons, neutrons, …)
  - -1 for all anti-baryons
  - 0   for photons, leptons and mesons
- The baryon number is additive and the net baryon number does not change in any process.
- The proton is the lightest baryon and so does not decay. However, proton decay is predicted at some small level by Grand Unified Theories.

Lepton Number

- The following reaction does not seem to occur, even at high energies:

$$e^- + e^- \nrightarrow \pi^- + \pi^-$$

- Define **lepton number**:
  - +1 for all leptons (e^-, μ^-, τ^-, neutrinos)
  - -1 for all anti-leptons (e^+, μ^+, τ^+, anti-neutrinos)
  - 0 for photon and hadrons
- Also, the following decay does not seem to occur:

$$\mu^- \nrightarrow e^- + \gamma$$

- Each type of lepton is separately conserved: electron-lepton #, muon-lepton # and tau-lepton #.

Lepton Summary

	(e^-, ν_e)	(μ^-, ν_μ)	(τ^-, ν_τ)
Electron # L_e	1	0	0
Muon # L_μ	0	1	0
τ-lepton # L_τ	0	0	1
$L = L_e + L_\mu + L_\tau$	1	1	1
Mass (MeV/c^2)	m_e =0.51	m_μ = 106	m_τ = 1777
Lifetime (s)	$\tau_e > 1.4 \times 10^{31}$	$\tau_\mu = 1.4 \times 10^{-6}$	$\tau_\tau = 2.9 \times 10^{-13}$

Strangeness

- Certain mesons (K) and baryons (Σ, Λ^0) are produced strongly (large cross sections of order mb), yet have lifetimes characteristic of weak decays (~10^{-10} s).
- The meson and baryon are always produced in pairs (K and Σ) or (K and Λ^0): *associated production*.
- Murray Gell-Mann and Abraham Pais proposed a **strangeness** quantum number:
  - Additive, like lepton # and baryon #.
  - Conserved in strong interactions (hence associated production).
  - Not conserved in weak decays.

Associated Production

- Strange baryons and mesons can be produced strongly, but only in pairs, conserving overall strangeness:

$$\pi^- + p \rightarrow \underbrace{K^0}_{S=1} + \underbrace{\Lambda^0}_{S=-1}, \text{ but } \pi^- + p \not\rightarrow \pi^0 + \Lambda^0$$

$$\pi^- + p \rightarrow \underbrace{K^+}_{S=1} + \pi^- + \underbrace{\Lambda^0}_{S=-1}, \text{ but } \pi^- + p \not\rightarrow \underbrace{K^-}_{S=-1} + \pi^+ + \underbrace{\Lambda^0}_{S=-1}$$

- The subsequent decays of the strange mesons and baryons are weak and do not conserve strangeness.

Strangeness Assignments

- By examining strong associated-production reactions, one can deduce that the strangeness of the K^+ and K^0 must be opposite to that of the Σ^+, Σ^0, Σ^- and Λ^0.
- Arbitrarily choose $S(K^0) = 1$. Then:
  - $S(K^+) = S(K^0) = 1$
  - $S(K^-) = S(\Lambda^0) = S(\Sigma^+) = S(\Sigma^0) = S(\Sigma^-) = -1$
  - $S(\Xi^0) = S(\Xi^-) = -2$
  - "Ordinary" mesons, baryons and the photon all have $S = 0$.
- Strangeness is only conserved in strong and EM interactions. Leptons decay weakly and so cannot be assigned strangeness.

Isospin

- Strong interactions do not distinguish between protons and neutrons.
  - *p-p*, *n-p* and *n-n* strong binding forces are the same.
  - *p+p* and *n+n* scattering are the same when corrected for Coulomb effects.
- Thus we can consider protons and neutrons to be different states of the isospin 1/2 **nucleon**:

$$p=\begin{pmatrix}1\\0\end{pmatrix} \text{ with } I_3=+\frac{1}{2}, \quad n=\begin{pmatrix}0\\1\end{pmatrix} \text{ with } I_3=-\frac{1}{2}$$

- This is analogous to spin-up and spin-down orientations of a spin-1/2 particle in the absence of a magnetic field.

Isospin, cont'd.

- If this symmetry were exact then the proton and neutron would have the same mass. This is called **isospin symmetry** and is a property of the strong interaction, but is violated by the weak and EM interactions.
- Another example: the pion. The pion is an **isovector** (isospin 1) particle (analogous to spin-1):

$$\pi^{+} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ with } I_3 = +1, \quad \pi^{0} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ with } I_3 = 0, \quad \pi^{-} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ with } I_3 = -1$$

- Isospin is conserved in strong interactions, but not weak or EM interactions. Since the photon and the leptons do not interact strongly, they cannot be assigned an isospin quantum number.

Gell-Mann-Nishijima Relation

- There is a relation between the various quantum numbers:

$$\boxed{Q = I_3 + \frac{Y}{2}} = I_3 + \frac{B+S}{2}$$

$$I_3 = 3^{\text{rd}} \text{ component of isospin}$$

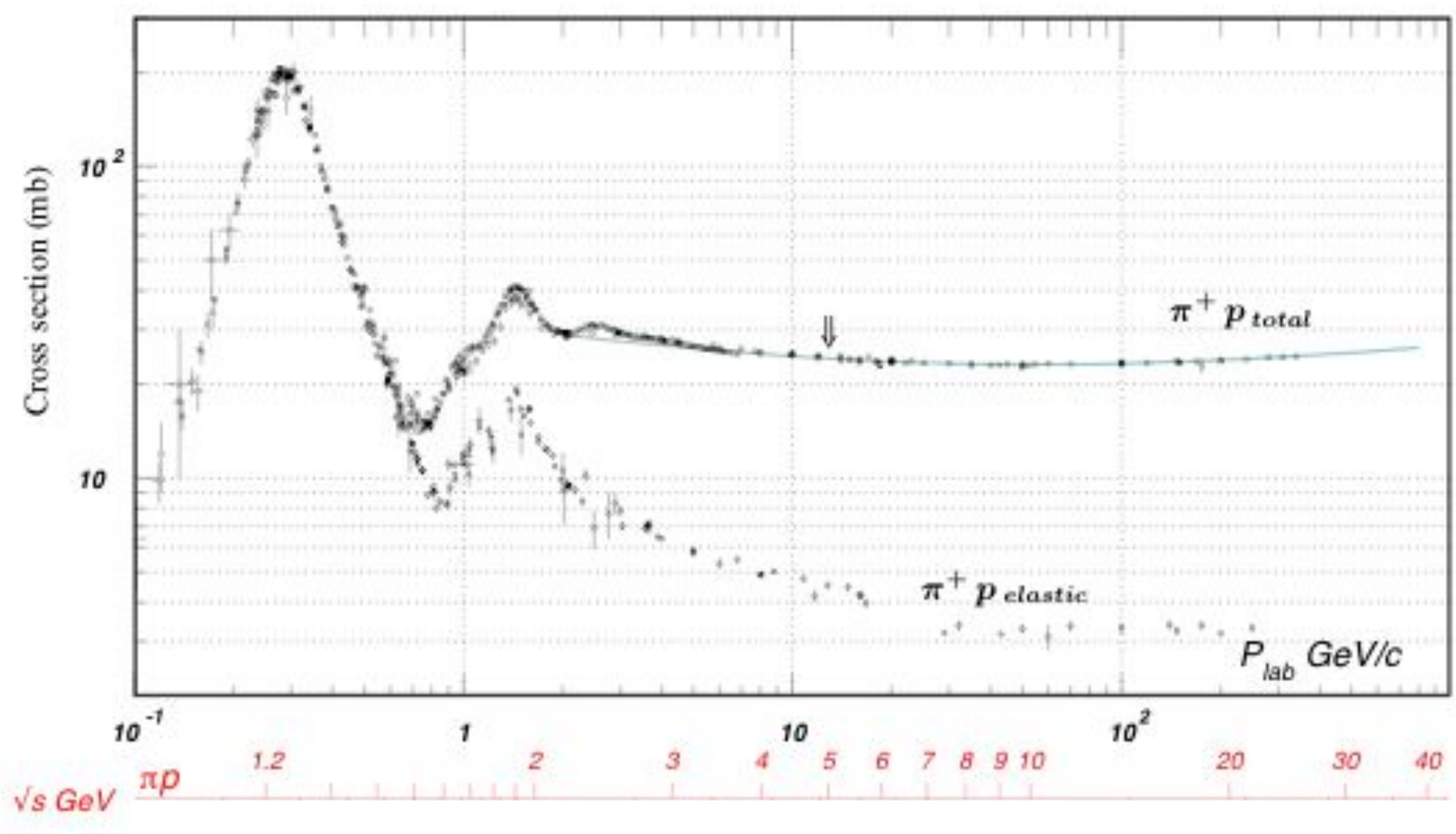
$$Y = B + S = \text{ strong hypercharge}$$

- Discovery of particles with new flavor quantum numbers (charm and bottom) requires redefining the hypercharge. If we include all these flavor quantum numbers in *Y*, the above shaded relation holds for all known particles.

Production and Decay of Resonances

- Nucleons and other hadrons, like nuclei, have excited states, or **resonances**.
- These states are unstable and decay quickly, but can be observed in two ways:
  - Formation, or *s-channel* studies. Here the resonance is observed as a peak in the scattering cross section vs. $\sqrt{s}$.
  - By detecting the decay products and measuring their invariant mass.

Pion-Proton Scattering



From http://pdg.lbl.gov/2005/reviews/hadronicrpp.pdf

The Delta Resonance

- The first excited state of the nucleon is the $\Delta(1232)$, i.e. an isospin 3/2 excitation (4 charge states) occurring at an invariant mass of 1232 MeV.
- The resonance, being short-lived, is measured to have a finite width, $\Gamma_\Delta c^2 \approx 100$ MeV (the full width at half maximum).
- This width corresponds to a lifetime:

$$\tau_\Delta \approx \frac{\hbar}{\Gamma_\Delta c^2} \approx \frac{6.6 \times 10^{-22}\ \text{MeV - sec}}{100\ \text{MeV}} \approx 10^{-23}\ \text{sec}$$

- The shape of the peak is described by a Lorentzian or *Breit-Wigner* form. Before discussing this, let's examine another way of discovering resonances …

Decay Channel Resonances

- For practical reasons, not all resonances can be found in this fashion (formation channel). For example, resonances among pions would require high-flux colliding pion beams.
- The ρ meson can be observed in:

$$\pi^- + p \to \rho^0 + n \to \pi^+ + \pi^- + n$$

- The intermediate state is not actually observed, but inferred from the invariant mass of the $\pi^+\pi^-$ system:

$$\left(M_\rho c^2\right)^2 = \left(E_\rho^2 - p_\rho^2 c^2\right) = \left(E_{\pi^+} + E_{\pi^-}\right)^2 - \left(\vec{p}_{\pi^+} + \vec{p}_{\pi^-}\right)^2 c^2 = s_{\pi\pi}$$

- The cross section vs. $s_{\pi\pi}$ peaks, indicating the presence of the resonance and establishing its mass.

Breit-Wigner Lineshape

- For a state of central mass M_0 and mean life $\hbar/(\Gamma c^2)$:

$$\psi(t) \propto \exp\left[-\frac{ic^2}{\hbar}\left(M_0 - i\frac{\Gamma}{2}\right)t\right], \quad t > 0$$

$$\text{and} \quad |\psi(t)|^2 \propto \exp\left(-\frac{\Gamma c^2 t}{\hbar}\right) \Rightarrow \quad \text{mean life} = \frac{\hbar}{\Gamma c^2}$$

- The Fourier transform of $\psi(t)$ is:

$$\psi(M) \propto \int_0^\infty dt\, \psi(t) e^{\frac{i}{\hbar}Mc^2 t} \propto \frac{1}{(M - M_0) + i\frac{\Gamma}{2}}$$

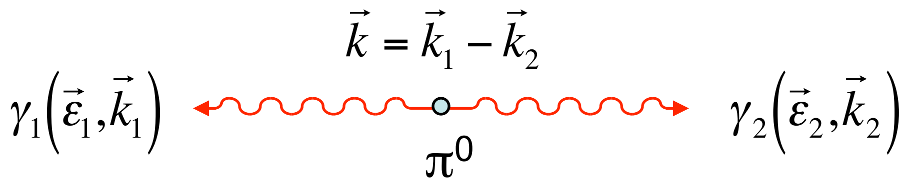
$$\text{and} \quad \boxed{|\psi(M)|^2 \propto \frac{1}{(M - M_0)^2 + \frac{\Gamma^2}{4}}}$$

Determining Spins

- There are various ways to determine the spin of a particle:
  - Stern-Gerlach experiment: electron and proton reveal two lines $\Rightarrow$ spin 1/2.
  - Angular momentum conservation: neutrino is spin 1/2.
  - Decay distributions of particles.
- The photon is the quantum of the electromagnetic field which is described by a vector potential $\Rightarrow$ photon is spin 1.
  - Normally, this would imply three possible angular momentum states. However, the electric and magnetic fields are perpendicular to the direction of propagation so there is no longitudinal degree of freedom for the photon.
  - The lack of a longitudinal polarization is related to the photon having zero rest mass.

Spin of π^0

In π^0 rest frame, the decay is back-to-back:



- Photons are identical bosons and so the final wave function must be symmetric under interchange.
- There are three vectors in this problem: $\vec{\varepsilon}_1,\ \vec{\varepsilon}_2,\ \vec{k}$
- The only scalar and vector quantities which can be constructed from these three vectors and which are symmetric under interchange of the photons are:

$$\vec{k} \times \left(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2\right),\ \ \vec{k} \cdot \left(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2\right),\ \text{ and }\ \vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2$$

Spin of π^0, cont'd.

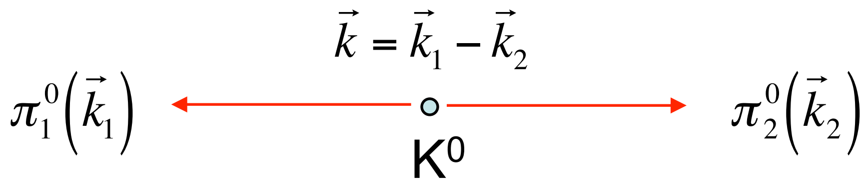
- But: $\vec{k} \times (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2) = 0$ since $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2 \perp \vec{k}$

  $\vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2 = 0$ since $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$ are seen to be orthogonal
- So the only possibility is: $\psi_{\text{final}} \propto \vec{k} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$
- The π^0 wave function must have such a component, or else the decay would not occur (zero overlap).
- So the π^0 is a scalar under rotations

  $\Rightarrow$ spin = 0

Spin of K^0

In K^0 rest frame, the decay is back-to-back:



- The pions are spin 0. So the angular momentum in the final state is just the orbital angular momentum of the two pions.
- The pions are identical bosons and so the wave function is symmetric under interchange.
- Interchange of the two pions is equivalent to a parity transformation. The symmetric requirement amounts to even parity $\Rightarrow \ell$ is even: 0, 2, 4, …

Spin of K^0, cont'd.

- Since the pions are spin 0, the only vector is $\vec{k}$
- Under exchange of the pions: $\vec{k} \rightarrow -\vec{k}$
- The wave function symmetry therefore implies:

$$\psi_{\text{final}} \propto \vec{k} \cdot \vec{k}$$

- The final wave function, and therefore the K^0 wave function is a scalar $\Rightarrow$ spin = 0.
- This is also demonstrated by the isotropic decay in the rest frame of the K^0.

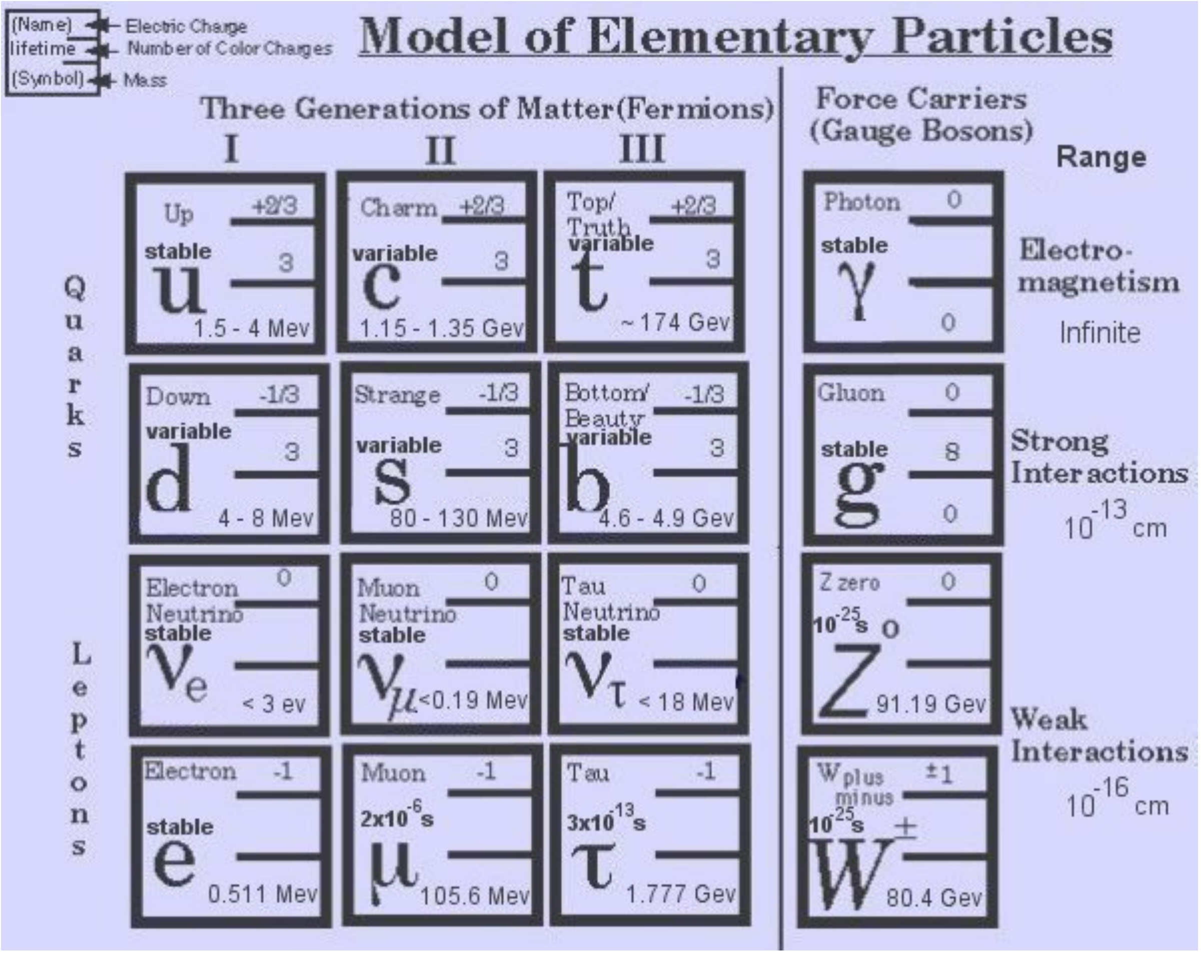
Violation of Quantum Numbers

- Strong decays conserve the various quantum numbers.
- Electromagnetic and weak decays violate some of the quantum numbers, i.e. those related to the quark **flavor**.
- Weak decays may be divided into:
  - *Hadronic*: Only hadrons in the final state.
  - *Semi-leptonic*: Some lepton(s) in the final state.
  - *Leptonic*: Only leptons in the final state.

Decays and Quantum Numbers

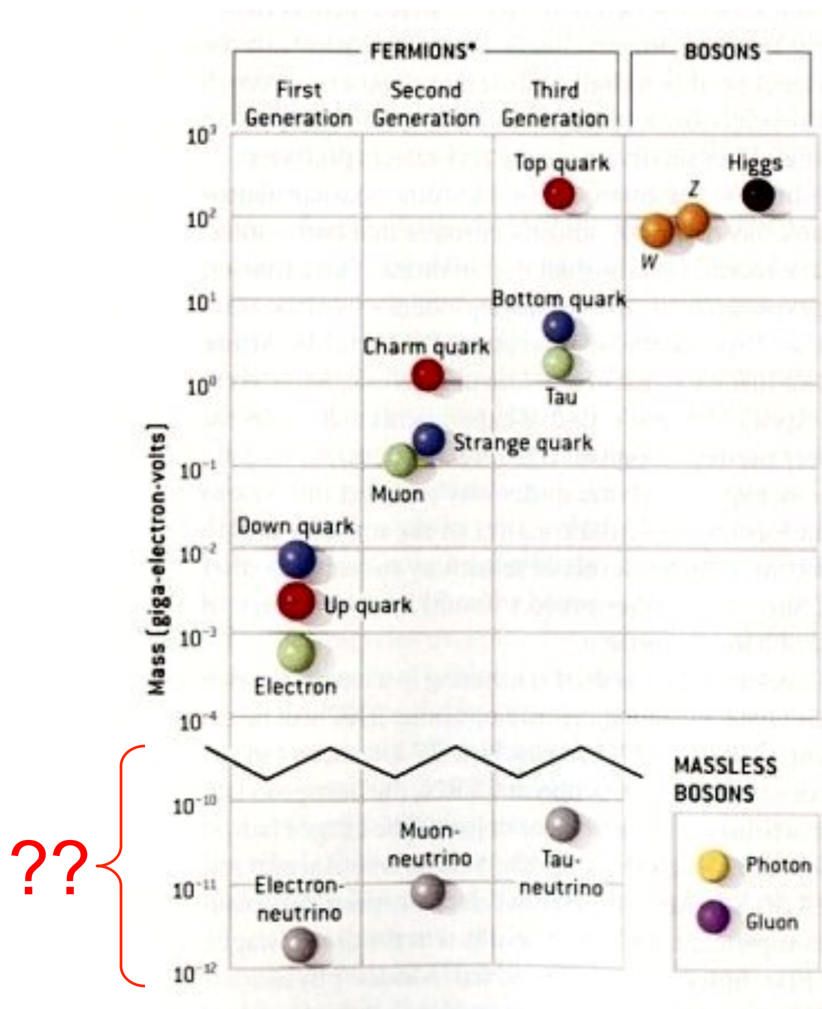
- Hadronic weak decays
  - $|\Delta S| = 1$, $|\Delta I_3| = 1/2$, $|\Delta I| = 1/2$
  - Rarely $|\Delta S| = 2$ or $|\Delta I| = 3/2$
- Semi-leptonic weak decays
  - Strangeness preserving: $|\Delta S| = 0$, $|\Delta I_3| = 1$, $|\Delta I| = 1$
  - Strangeness changing: $|\Delta S| = 1$, $|\Delta I_3| = 1/2$ or $|\Delta I| = 1/2$
  - Rarely: $|\Delta I| = 3/2$
- EM decays
  - $|\Delta S| = 0$, $|\Delta I_3| = 0$, $|\Delta I| = 1$ or 0

Summary of Elementary Particles



From http://universe-review.ca/I15-02-elementary.jpg

Particle Masses



Adapted from http://universe-review.ca/I15-21-fermiontable.jpg

Summary of Interactions

Interaction	Gravitational	Weak	Electromagnetic	Strong: Fundamental	Strong: Residual
		Electroweak	Electroweak		
Acts on:	Mass-Energy	Flavor charge	Electric charge	Color charge	See info.
Particles Experiencing it:	All	Leptons Quarks	Electrically-charged	Quarks Gluons	Hadrons
Particles Carrying it:	Graviton (not yet observed)	$W^+ W^- Z^0$	γ	Gluons	Mesons
Strength for: 2 quarks at 10^{-18} m	10^{-41}	0.8	1	25	Not applicable to quarks
Strength for: 2 quarks at 3×10^{-17} m (relative to e/m)	10^{-41}	10^{-4}	1	60	Not applicable to quarks
2 protons in nucleus at 10^{-14} m	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

From http://universe-review.ca/I15-02-interactions.gif