415 Midterm (20 points) March 6 2014, 16:15 ÷ 17:45.

Problem 1. Find maximal scattering angle for elastic collision of deuteron off the proton target. The proton is initially at rest. Assume non-relativistic kinematics.

Hint: consider the problem in c.m. frame and use Eq. (1.53) from the textbook

Solution

Eq. (1.53) is

$$\cos\theta_{\rm Lab} \;=\; \frac{\cos\theta_{\rm c.m.} + r}{\sqrt{1 + 2r\cos\theta_{\rm c.m.} + r^2}}$$

where $\frac{m_1}{m_2} = 2$. Differentiating with respect to $\cos\theta_{\rm c.m.}$ we see that the maximum occurs at $\cos\theta_{\rm c.m.} = -\frac{1}{r}$ which corresponds to

$$\cos\theta_{\rm Lab} \;=\; \frac{\sqrt{r^2-1}}{r} \;=\; \frac{\sqrt{3}}{2} \qquad \Rightarrow \qquad \theta^{\rm max}_{\rm Lab} \;=\; 30^o$$

Problem 2.

The ground states of "mirror nuclei" $^{27}{\rm Si}^{14}$ and $^{27}{\rm Al}^{13}$ are identical except for charge. If their mass difference is 6 MeV, estimate their radius (neglecting the proton-neutron mass difference).

Hint: electrostatic energy of a charge Q distributed uniformly throughout a sphere of radius R is $\frac{3Q^2}{5R}$ in SGS or $\frac{3Q^2}{20\pi\epsilon_0 R}$ in SI system.

Solution

The mass difference between the mirror nuclei $^{27}{\rm Si}^{14}$ and $^{27}{\rm Al}^{13}$ can be considered as due to the difference in electrostatic energy:

$$\Delta W \;=\; \frac{3e^2}{20\pi\epsilon_0 R}(Z_1^2 - Z_2^2)$$

Thus,

$$R \;=\; \frac{3e^2}{20\pi\epsilon_0 \Delta W}(14^2 - 13^2) \;=\; 3.88\times 10^{-15}{\rm m} \;=\; 3.88{\rm fm}$$

Problem 3.

The ratio of number of U^{238} nuclei to that of Pb^{206} nuclei in a certain ore is 2.8. Find the age of the ore assuming that all of Pb^{206} is a product of U^{238} decay. The half-time of U^{238} is 4.5×10^9 years

Solution

Denote the initial number of U^{238} nuclei by N_0. The amount of uranium nuclei after time t is $N = N_0 e^{-\lambda t}$ and hence the amount of lead nuclei is $N_l = N_0(1 - e^{-\lambda t})$. We know that

$$\frac{N(t)}{N_l(t)} = n = 2.8 \quad \Leftrightarrow \quad \frac{e^{-\lambda t}}{1 - e^{-\lambda t}} = n \quad \Rightarrow \; t = \frac{1}{\lambda} \ln \frac{1+n}{n} \;=\; \frac{0.3053}{\lambda}$$

From Eq. (5.24) $t_{1/2} = \frac{\ln 2}{\lambda}$ so

$$t \;=\; \frac{0.3053}{0.693} t_{1/2} \;=\; \frac{0.3053}{0.693} \times 4.5 \times 10^9 \text{yrs} \;=\; 1.98 \times 10^9 \text{yrs}$$

Problem 4.

After 3 α - decays and two β-decays, the ${}^{92}U^{238}$ isotope turns to ${}^{?}?^{?}$

Solution

${}^{88}Ra^{226}$

Problem 5. A typical energy for X-rays used in dentistry is 50 keV. At this energy the cross section (mainly due to absorption) is about 2840 barn. Find how thick should be the lead sheet to be able to absorb 99% of such radiation. Density of the lead is 11.4gm/cm^3.

Solution

From Eq. (6.28)

$$\mu \;=\; \rho \frac{A_0}{A} \sigma \;=\; 11.3 \frac{g}{cm^3} \frac{6.02 \times 10^{23}}{207.2g} 2.84 \times 10^{-21} cm^2 \;=\; 93.2 cm^{-1}$$

From Eq. (6.18)

$$I \;=\; I_0 e^{-\mu l} \quad \Rightarrow \quad l = \frac{1}{\mu} \ln \frac{I_0}{I} \;=\; \frac{1}{\mu} \ln 100 \;=\; 0.5\text{mm}$$

All problems have equal weight.