PROBLEM 3.8 In Example 3.2. we assumed that the conducting sphere was grounded ($V = 0$). But with the addition of a second image charge, the same basic model will handle the case of a sphere at **any** potential V_0 (relative, of course to infinity).

What charge should you use, and where should you put it?

Find the force of attraction between a point charge q and a **neutral** conducting sphere.

<u>**Known:**</u> $q,\ R,\ a$

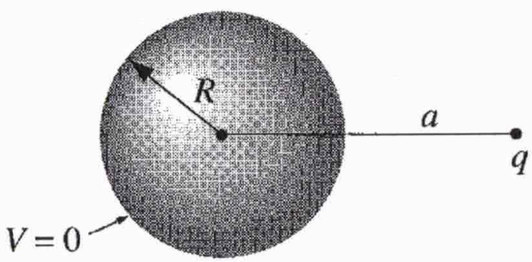


Figure 3.12

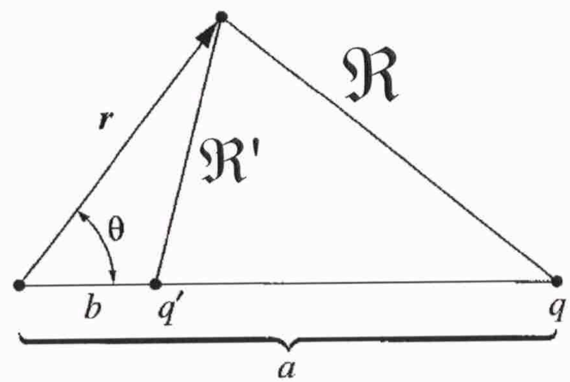


Figure 3.13

PROBLEM 3.8

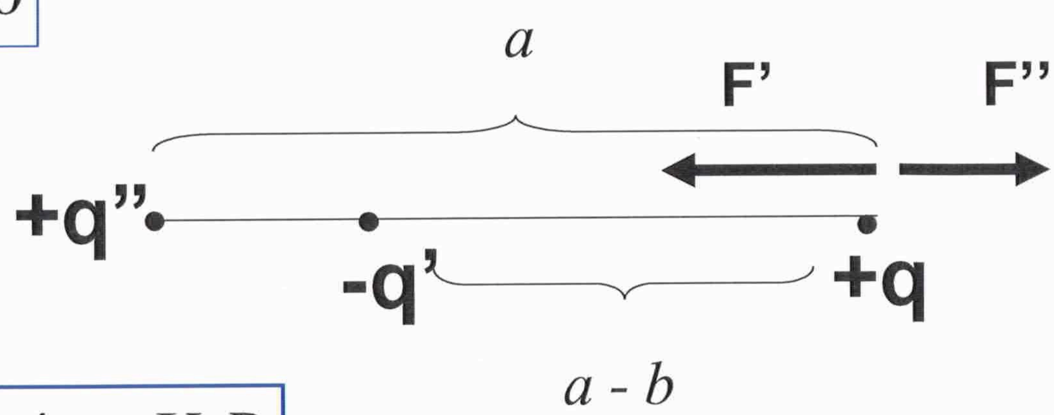
IDENTIFY Relevant concepts

- Equipotential surfaces
- Method of images
- Potential $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Neutral sphere: $q' + q'' = 0$

SET UP

Draw diagram:



EXECUTE

$$q'' = 4\pi\varepsilon_0 V_0 R$$

V_0 is a new, non-zero potential of the sphere, which is an equipotential surface.

Also,

$$q'' = -q' = \frac{R}{a}q \quad and \quad b = \frac{R^2}{a}$$

$$\vec{F} = \vec{F}' + \vec{F}'' \quad \Rightarrow \quad F = F'' - F'$$

PROBLEM 3.8 (cont.)

EXECUTE (cont.)

$$F=\frac{1}{4\pi\varepsilon_0}q\left(\frac{q''}{a^2}+\frac{q'}{(a-b)^2}\right)=\frac{qq'}{4\pi\varepsilon_0}\left(-\frac{1}{a^2}+\frac{1}{(a-b)^2}\right)$$

$$=\frac{qq'}{4\pi\varepsilon_0}\frac{-\not{a}^2+2ab-b^2+\not{a}^2}{a^2(a-b)^2}=\frac{qq'}{4\pi\varepsilon_0}\frac{b(2a-b)}{a^2(a-b)^2}$$

$$=\frac{q\left(-\frac{Rq}{a}\right)}{4\pi\varepsilon_0}\frac{\left(\frac{R^2}{a}\right)\left(2a-\frac{R^2}{a}\right)}{a^2\left(a-\frac{R^2}{a}\right)^2}=-\frac{q^2}{4\pi\varepsilon_0}\frac{\left(\frac{R}{a}\right)\left(\frac{R^2}{a^2}\right)\left(2a^2-R^2\right)}{\left(a^2-R^2\right)^2}$$

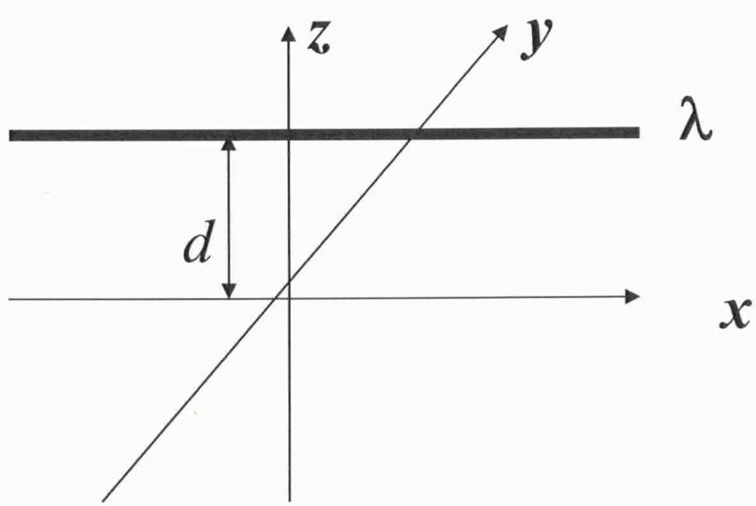
$$F=-\frac{q^2}{4\pi\varepsilon_0}\left(\frac{R}{a}\right)^3\frac{2a^2-R^2}{\left(a^2-R^2\right)^2}.$$

PROBLEM 3.9

A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to x-axis and directly above it, and the conducting plane is the xy plane.)

(a) Find the potential in the region above the plane. [**Hint:** Refer to problem 2.47.]

(b) Find the charge density σ induced on the conducting plane.



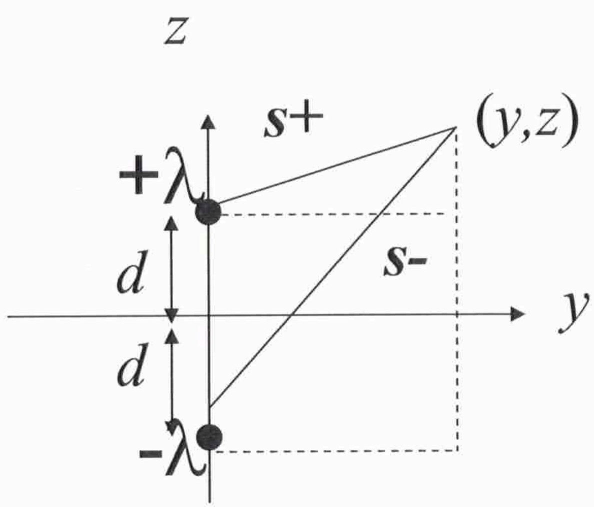
PROBLEM 3.9 (a)

IDENTIFY Relevant concepts

- Method of images
- Potential due to two line charges from Problem 2.47 (See lectures, Ch. 2)

SET UP

Draw diagram:



Conductive plane is replaced by a mirror line charge $-\lambda$ (see diagram).
Problem is now equivalent to 2.47 (a).

EXECUTE

$$V(y,z) = \frac{2\lambda}{4\pi\varepsilon_0}\ln(s_-/s_+) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(s_-^2/s_+^2\right)$$

$$\boxed{V(y,z) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{y^2+(z+d)^2}{y^2+(z-d)^2}\right)}$$

PROBLEM 3.9 (b)

IDENTIFY Relevant concepts

$$xy-plane \ \ at \ \ z=0:$$

$$\sigma(y) = -\varepsilon_0 \frac{\partial V}{\partial n} \qquad \frac{\partial V}{\partial n} = \left.\frac{\partial V}{\partial z}\right|_{z=0}$$

EXECUTE

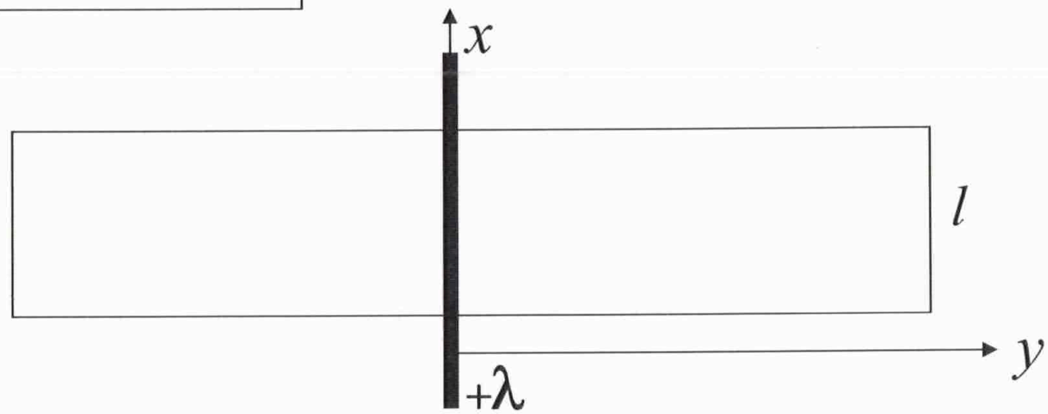
$$\sigma(y) = \left.\frac{\partial V}{\partial z}\right|_{z=0} = -\varepsilon_0 \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{1}{y^2+(z+d)^2}2(z+d) - \frac{1}{y^2+(z-d)^2}2(z-d)\right\}\Bigg|_{z=0}$$

$$= -\frac{2\lambda}{4\pi}\left\{\frac{d}{y^2+d^2} - \frac{-d}{y^2+d^2}\right\} = -\frac{\lambda d}{\pi\left(y^2+d^2\right)}$$

PROBLEM 3.9 (b)

EVALUATE

Consider a strip of width l in xy–plane, perpendicular to y.



Induced charge on the strip is:

$$q_{ind} = \int_{-\infty}^{\infty} \sigma(y) \cdot \underbrace{l dy}_{dA} = -\frac{l\lambda d}{\pi} \int_{-\infty}^{\infty} \frac{1}{y^2 + d^2} dy = -\frac{l\lambda d}{\pi} \int_{-\infty}^{\infty} \frac{1}{d} \frac{1}{\left(\frac{y}{d}\right)^2 + 1} d\left(\frac{y}{d}\right)$$

$$= -\frac{l\lambda d}{\pi}\left[\frac{1}{d}\tan^{-1}\left(\frac{y}{d}\right)\right]_{-\infty}^{\infty} = -\frac{l\lambda}{\pi}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = -\lambda l \Rightarrow \boxed{\frac{q_{ind}}{l} = -\lambda}$$

PROBLEM 3.14

A rectangular pipe, running parallel to the z-axis $\left(from -\infty\ to +\infty\right)$, has three grounded metal sides, at $y=0$, $y=a$, and $x=0$. The fourth side, at $x=b$, is maintained at a specified potential $V_0(y)$.

(a) Develop a general formula for the potential within the pipe.

(b) Find the potential explicitly, for the case $V_0(y)=V_0$ (a constant).

PROBLEM 3.14a,b

IDENTIFY relevant concepts

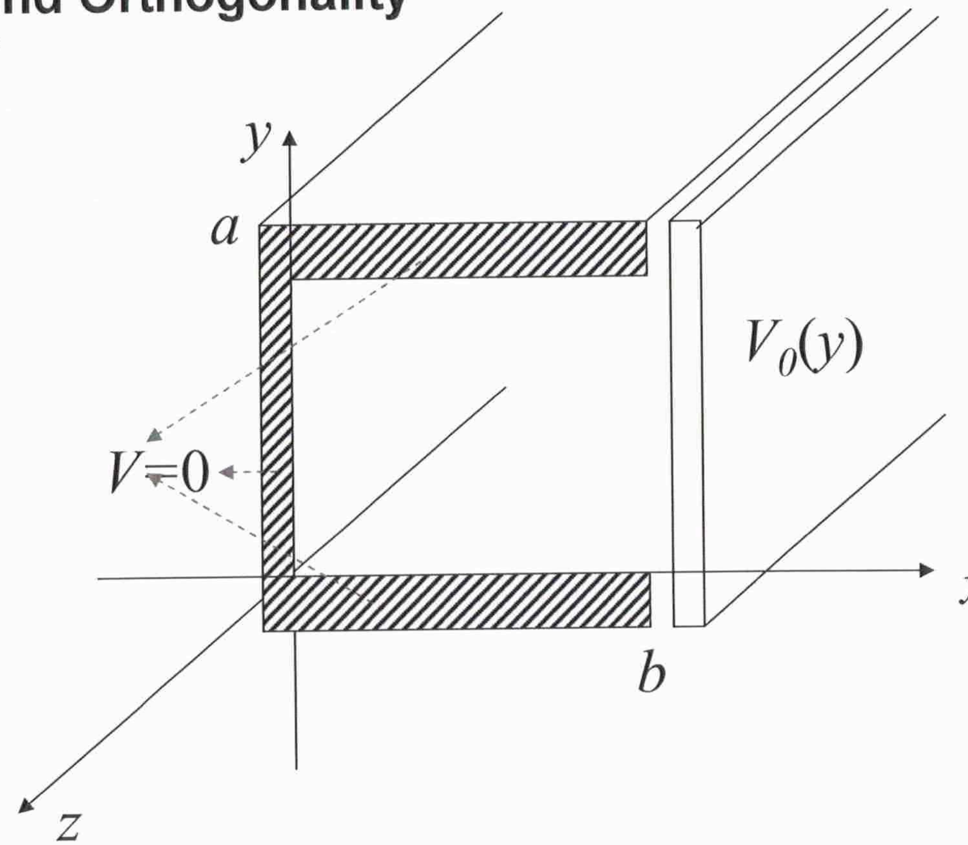
- Laplace's Equation
- Boundary conditions

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$-\infty < z < \infty \quad \Downarrow \quad V \neq V(x, y, z)$$

- Separation of variables (Examples 3.3 and 3.4)
- Completeness and Orthogonality
- "Fourier's trick"

SET UP Draw diagram



Write Boundary Conditions:

$(i) \quad V(x,0) = 0$

$(ii) \quad V(x,a) = 0$

$(iii) \quad V(0,y) = 0$

$(iv) \quad V(b,y) = V_0(y)$

PROBLEM 3.14a

EXECUTE

From *Eqs.* 3.25-27:

$$V(x,y)=\left(Ae^{kx}+Be^{-kx}\right)\left(C\sin ky+D\cos ky\right)$$

Apply the boundary conditions:

$$(i)\quad y=0, V=0, \sin ky=0 \Rightarrow \boxed{D=0}$$

$$(ii)\quad y=a, V=0, \Rightarrow \sin ka=0 \therefore \boxed{ka=n\pi} \quad k=\frac{n\pi}{a}$$

$$(iii)\quad x=0, e^{kx}=e^{-kx}=1, V=0 \Rightarrow A+B=0 \therefore \boxed{A=-B}$$

$$V(x,y)=AC\left(e^{n\pi x/a}-e^{-n\pi x/a}\right)\sin(n\pi y/a)$$

$$e^{n\pi x/a}-e^{-n\pi x/a}=2\sinh(n\pi x/a)$$

$$\Rightarrow V(x,y)=2AC\sinh(n\pi x/a)\sin(n\pi y/a).$$

Apply the completeness principle:

$$V(x,y)=\sum_{n=1}^{\infty}C_n\sinh(n\pi x/a)\sin(n\pi y/a)$$

is complete on the interval $0\leq y\leq a$

EXECUTE

PROBLEM 3.14a (cont.)

Apply boundary condition (iv):

$$(iv) \quad \sum_{n=1}^{\infty} C_n \sinh(n\pi b/a)\sin(n\pi y/a) = V_0(y)$$

Apply "Fourier's trick"

$$\sin(n'\pi y/a)\sum_{n=1}^{\infty} C_n \sinh(n\pi b/a)\sin(n\pi y/a) = V_0(y)\sin(n'\pi y/a)$$

$$\sum_{n=1}^{\infty} C_n \sinh(n\pi b/a)\int_0^a \sin(n'\pi y/a)\sin(n\pi y/a)dy = \int_0^a V_0(y)\sin(n'\pi y/a)dy$$

Use the orthogonality property of the complete set of solutions:

$$\int_0^a \sin(n'\pi y/a)\sin(n\pi y/a)dy = \begin{cases} 0 & n' \neq n \\ \dfrac{a}{2} & n' = n \end{cases} \qquad (Eq.\ 3.33)$$

$$\Rightarrow \boxed{C_n = \frac{2}{a\sinh(n\pi b/a)}\int_0^a V_0(y)\sin(n\pi y/a)dy}$$

PROBLEM 3.14b $V_0(y)=V_0$

EXECUTE

$$C_n = \frac{2}{a\sinh(n\pi b/a)} V_0 \int_0^a \sin(n\pi y/a)\,dy$$

$$= \frac{2}{a\sinh(n\pi b/a)} V_0 \frac{a}{n\pi}\left(-\cos(n\pi y/a)\right)\Big|_0^a =$$

$$= \begin{cases} 0 & n = 0,2,4,\ldots \\ \dfrac{4V_0}{n\pi\sinh(n\pi b/a)} & n = 1,3,5,\ldots \end{cases}$$

$$\Rightarrow V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,..} \frac{\sinh(n\pi x/a)\sin(n\pi y/a)}{n\sinh(n\pi b/a)}.$$

PROBLEM 3.18

The potential at the surface of a sphere (radius R) is given by

$$V_0 = k\cos 3\theta$$

where k is a constant. Find the potential inside and outside the sphere, as well as the surface charge density $\sigma(\theta)$ on the sphere. (Assume there is no charge inside and outside the sphere.)

PROBLEM 3.18

IDENTIFY relevant concepts

Spherical coordinates:

Express potential in the terms of Legendre's polynomials:

$$V(r,\theta)=\sum_{l=0}^{\infty}\left(A_l r^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta) \qquad (Eq.\,3.65)$$

Inside the sphere (Example 3.6) $V(0,\theta)\neq\infty \quad\Rightarrow\quad B_l=0$

$$\Rightarrow\quad V(r,\theta)=\sum_{l=0}^{\infty}A_l r^l P_l(\cos\theta), \qquad r\le R$$

Outside the sphere (Example 3.7) $r\to\infty,\quad V\to 0 \quad\Rightarrow\quad A_l=0$

$$\Rightarrow\quad V(r,\theta)=\sum_{l=0}^{\infty}\frac{B_l}{r^{l+1}}P_l(\cos\theta), \qquad r\ge R$$

Properties of Legendre's polynomials ***Trigonometric relations**

any polynomial of 3rd order

$$a_3x^3+a_2x^2+a_1x+a_0=\alpha P_3(x)+\beta P_2(x)+\gamma P_1(x)+\delta P_0(x)$$

PROBLEM 3.18 (cont.)

IDENTIFY relevant concepts (continued)

Properties of Legendre's polynomials:

***Orthogonality** ($Eq.\ 3.68$)

$$\int_0^{\pi} P_l(\cos\theta) P_{l'}(\cos\theta)\sin\theta\, d\theta = \frac{2}{2l+1}\delta_{ll'} = \begin{Bmatrix} 0 & l \neq l' \\ \dfrac{2}{2l+1} & l = l' \end{Bmatrix}$$

Potential is continuous at $r = R$ **(see Example 3.9)**

$$\sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta) = \sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta) \qquad (Eq.\ 3.80)$$

$$\Rightarrow \quad B_l = A_l R^{2l+1} \qquad (Eq.\ 3.81)$$

Radial derivative of potential is discontinuous at $r = R$ (see Example 3.9)

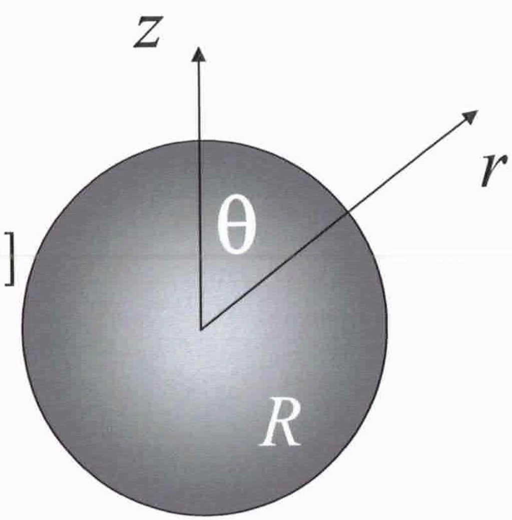
$$\left.\left(\frac{\partial V_{out}}{\partial r} - \frac{\partial V_{in}}{\partial r}\right)\right|_{r=R} = -\frac{\sigma_0(\theta)}{\varepsilon_0}$$

$$\Rightarrow \quad \sum_{l=0}^{\infty} (2l+1) A_l R^{l-1} P_l(\cos\theta) = \frac{\sigma_0(\theta)}{\varepsilon_0} \qquad (Eq.\ 3.83)$$

PROBLEM 3.18 (cont.)

SET UP

Draw diagram:



Make trigonometric transformations:

$$V_0(\theta) = k\cos(3\theta) = k\cos(2\theta+\theta) = k[\cos(2\theta)\cos\theta - \sin(2\theta)\sin\theta]$$

$$= k\left[(2\cos^2\theta - 1)\cos\theta - 2(1-\cos^2\theta)\cos\theta\right]$$

$$= k\left[4\cos^3\theta - 3\cos\theta\right]$$

EXECUTE

Express potential in terms of Legendre's polynomials (only odd-power terms are non-zero):

$$V_0(\theta) = k\left[4\cos^3\theta - 3\cos\theta\right] = k[\alpha P_3(\cos\theta) - \gamma P_1(\cos\theta)]$$

$$4\cos^3\theta - 3\cos\theta = \alpha\left[\frac{1}{2}(5\cos^3\theta - 3\cos\theta)\right] + \gamma\cos\theta =$$

$$\left(4 = \frac{5}{2}\alpha \quad \alpha = \frac{8}{5}\right)$$

$$= \frac{5\alpha}{2}\cos^3\theta - \left(\frac{3\alpha}{2} - \gamma\right)\cos\theta$$

$$\left(3 = \frac{3}{2}\frac{8}{5} - \gamma \quad \gamma = -\frac{3}{5}\right)$$

$$\Rightarrow \boxed{\alpha = \frac{8}{5}, \quad \gamma = \frac{24}{10} - 3 = -\frac{3}{5}}$$

$$\Rightarrow \boxed{V_0(\theta) = \frac{k}{5}[8P_3(\cos\theta) - 3P_1(\cos\theta)]}$$

PROBLEM 3.18 (cont.)

EXECUTE (cont.)

$r \le R$

$$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta),$$

$$A_l = \frac{2l+1}{2R^l}\int_0^{\pi} V_0(\theta) P_l(\cos\theta)\sin\theta d\theta \qquad (Eq.\,3.69)$$

$$A_l = \frac{(2l+1)}{2R^l}\frac{k}{5}\left\{8\int_0^{\pi} P_3(\cos\theta)P_l(\cos\theta)\sin\theta d\theta - 3\int_0^{\pi} P_1(\cos\theta)P_l(\cos\theta)\sin\theta d\theta\right\}$$

Apply orthogonality of Legendre's polynomials:

$$A_l = \frac{k}{5}\frac{(2l+1)}{2R^l}\left\{8\frac{2}{2l+1}\delta_{l3} - 3\frac{2}{2l+1}\delta_{l1}\right\} = \frac{k}{5}\frac{1}{R^l}\{8\delta_{l3} - 3\delta_{l1}\}$$

$$A_1 = -\frac{3k}{5R}, \quad A_3 = \frac{8k}{5R^3} \qquad A_l = 0 \quad otherwise$$

PROBLEM 3.18 (cont.)

EXECUTE (cont.)

$r \leq R$

$$V(r,\theta) = -\frac{3k}{5R} r P_1(\cos\theta) + \frac{8k}{5R^3} r^3 P_3(\cos\theta)$$

$$= \frac{k}{5}\left[8\left(\frac{r}{R}\right)^3 P_3(\cos\theta) - 3\left(\frac{r}{R}\right) P_1(\cos\theta)\right]$$

$$= \frac{k}{5}\left\{8\left(\frac{r}{R}\right)^3 \frac{1}{2}\left(5\cos^3\theta - 3\cos\theta\right) - 3\left(\frac{r}{R}\right)\cos\theta\right\}$$

$$V(r,\theta) = \frac{k}{5}\left(\frac{r}{R}\right)\cos\theta\left\{4\left(\frac{r}{R}\right)^2\left(5\cos^2 - 3\right) - 3\right\}$$

$r \geq R$ **From continuity of V at R ($r=R$):**

$$B_l = A_l R^{2l+1} = \begin{cases} \dfrac{8k}{5R^3} R^7 = \dfrac{8kR^4}{5} & if \quad l = 3 \\ -\dfrac{3k}{5R} R^3 = -\dfrac{3kR^2}{5} & if \quad l = 1 \end{cases} \quad (B_l = 0 \quad otherwise)$$

PROBLEM 3.18 (cont.)

EXECUTE (cont.)

$r \geq R$

$$V(r,\theta)=\sum_{l=0}^{\infty}\frac{B_l}{r^{l+1}}P_l(\cos\theta)$$

$$V(r,\theta)=-\frac{3kR^2}{5}\frac{1}{r^2}P_1(\cos\theta)+\frac{8kR^4}{5}\frac{1}{r^4}P_3(\cos\theta)$$

$$V(r,\theta)=\frac{k}{5}\left[8\left(\frac{R}{r}\right)^4 P_3(\cos\theta)-3\left(\frac{R}{r}\right)^2 P_1(\cos\theta)\right]$$

$$V(r,\theta)=\frac{k}{5}\left(\frac{R}{r}\right)^2\cos\theta\left\{4\left(\frac{R}{r}\right)^2\left(5\cos^2\theta-3\right)-3\right\}$$

Charge density

$$\frac{\sigma_0(\theta)}{\varepsilon_0}=\sum_{l=0}^{\infty}(2l+1)A_l R^{l-1}P_l(\cos\theta)$$

$$\sigma_0(\theta)=\varepsilon_0\left[3A_1P_1(\cos\theta)+7A_3R^2P_3(\cos\theta)\right]$$

PROBLEM 3.18 (cont.)

EXECUTE (cont.)

Charge density

$$\sigma_0(\theta) = \varepsilon_0\left[3\left(-\frac{3k}{5R}\right)P_1(\cos\theta) + 7\left(\frac{8k}{5R^3}\right)R^2 P_3(\cos\theta)\right]$$

$$= \frac{\varepsilon_0 k}{5R}\left[-9P_1(\cos\theta) + 56P_3(\cos\theta)\right]$$

$$= \frac{\varepsilon_0 k}{5R}\left[-9\cos\theta + 28\left(5\cos^3\theta - 3\cos\theta\right)\right]$$

$$\sigma_0(\theta) = \frac{\varepsilon_0 k}{5R}\cos\theta\left(140\cos^2\theta - 93\right)$$

PROBLEM 3.26

A sphere of radius R, centered at the origin, carries charge density

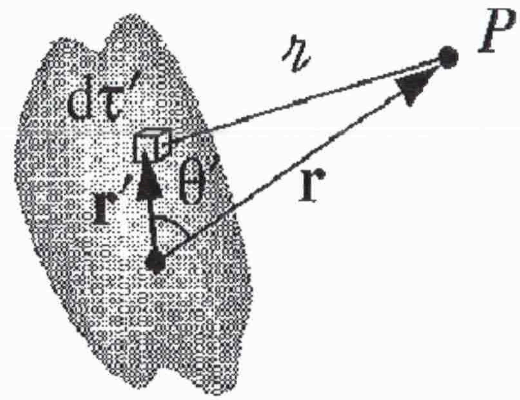
$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta$$

where k is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential for points on the z axis far from the sphere.

PROBLEM 3.26

IDENTIFY relevant concepts

Multipole expansion:



$$V(\vec{r}) = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int \rho(\vec{r}')d\tau'}_{monopole} + \ldots$$

$$\ldots + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int r'\cos\theta'\rho(\vec{r}')d\tau'}_{dipole} + \ldots$$

$$\ldots + \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\int (r')^2\left(\frac{3}{2}\cos^2\theta' - \frac{1}{2}\right)\rho(\vec{r}')d\tau' + \ldots}_{quadrupole} \qquad (Eq. 3.96)$$

Volume element:

$$d\tau = r^2 \sin\theta dr d\theta d\phi$$

PROBLEM 3.26 (cont.)

SET UP and EXECUTE **Monopole term:**

$$V_{monopole}(\vec{r}) = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{1}{r}\int\rho(\vec{r}\,')d\tau'}_{monopole} = \frac{Q}{4\pi\varepsilon_0 r},$$

$$Q = \int\rho(\vec{r}\,')d\tau' = kR\int\left[\frac{1}{r^2}(R-2r)\sin\theta\right]r^2\sin\theta dr d\theta d\phi$$

$$Q = kR\int_0^{2\pi}d\phi\int_0^{\pi}\sin^2\theta d\theta\int_0^{R}(R-2r)dr = 4\pi kR\left(Rr - r^2\right)\Big|_o^R = 4\pi kR\left(R^2 - R^2\right) = 0$$

$$\boxed{V_{monopole}(\vec{r}) = 0}$$

Dipole term:

$$V_{dipole}(\vec{r}) = \underbrace{\frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\int r'\cos\theta'\rho(\vec{r}\,')d\tau'}_{dipole} = \frac{1}{4\pi\varepsilon_0 r^2}I$$

PROBLEM 3.26 (cont.)

EXECUTE (cont.)

Dipole term:

$$I = \int r\cos\theta\rho d\tau = kR\left(\int_0^{2\pi} d\phi\right)\left(\int_0^{\pi}\sin^2\theta\cos\theta d\theta\right)\left[\int_0^{R} r(R-2r)dr\right]$$

$$\int_0^{\pi}\sin^2\theta\cos\theta d\theta = \frac{\sin^3\theta}{3}\Bigg|_0^{\pi} = \frac{1}{3}(0-0) = 0 \quad\Rightarrow\quad I = 0, \quad \boxed{V_{dipole} = 0}$$

Quadrupole term:

$$V_{quad} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\int (r')^2\left(\frac{3}{2}\cos^2\theta' - \frac{1}{2}\right)\rho(\vec{r}')d\tau' = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}I_q$$

$$I_{quad} = \int r^2\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\rho(\vec{r})d\tau$$

$$I_{quad} = \frac{kR}{2}\iiint r^2(3\cos^2\theta - 1)\left[\frac{1}{r^2}(R-2r)\sin\theta\right]r^2\sin\theta dr d\theta d\phi$$

$$\boxed{I_{quad}} = \frac{kR}{2}\underbrace{\left[\int_0^{2\pi} d\phi\right]}_{I_\phi}\underbrace{\left[\int_0^{\pi}(3\cos^2\theta - 1)\sin^2\theta d\theta\right]}_{I_\theta}\underbrace{\left[\int_0^{R} r^2(R-2r)dr\right]}_{I_r} = \boxed{\frac{kR}{2}I_\phi I_\theta I_r}$$

PROBLEM 3.26 (cont.)

EXECUTE (cont.) **Quadrupole term:**

$$I_r = \int_0^R r^2(R-2r)dr = \frac{r^3R}{3} - \frac{r^4}{2}\bigg|_0^R = \frac{R^4}{3} - \frac{R^4}{2} = \boxed{-\frac{R^4}{6}}$$

$$I_\theta = \int_0^\pi (3\cos^2\theta - 1)\sin^2\theta d\theta = \int_0^\pi (2 - 3\sin^2\theta)\sin^2\theta d\theta = 2\int_0^\pi \sin^2\theta d\theta - 3\int_0^\pi \sin^4\theta d\theta$$

From Tables of Integrals:

$$\int \sin^2\theta d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}$$

$$\int \sin^4\theta d\theta = \frac{\sin^3\theta\cos\theta}{4} + \frac{3}{4}\int \sin^2\theta d\theta = \frac{3\theta}{8} + \frac{\sin^3\theta\cos\theta}{4} - \frac{3\sin 2\theta}{16}$$

$$\Rightarrow \quad I_\theta = 2\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\bigg|_0^\pi - 3\left(\frac{3\theta}{8} + \frac{\sin^3\theta\cos\theta}{4} - \frac{3\sin 2\theta}{16}\right)\bigg|_0^\pi$$

$$I_\theta = 2\left(\frac{\pi}{2}\right) - 3\left(\frac{3\pi}{8}\right) = \pi\left(1 - \frac{9}{8}\right) = \boxed{-\frac{\pi}{8}}$$

$$I_\phi = \int_0^{2\pi} d\phi = \boxed{2\pi}$$

PROBLEM 3.26 (cont.)

EXECUTE (cont.)

$$I_{quad} = \frac{kR}{2} I_\phi I_\theta I_r = \frac{kR}{2}(2\pi)\left(-\frac{\pi}{8}\right)\left(-\frac{R^4}{6}\right) = \boxed{\frac{k\pi^2 R^5}{48}}$$

$$\boxed{V_{quad}(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\frac{k\pi^2 R^5}{48}}$$

(a) $r = 2R$

(b) $r = z$

$$V(r) = 0 + 0 + \frac{1}{4\pi\varepsilon_0}\frac{1}{z^3}\frac{k\pi^2 R^5}{48} + \ldots \cong \frac{1}{4\pi\varepsilon_0}\frac{1}{z^3}\frac{k\pi^2 R^5}{48}$$

$$r \rightarrow z, \quad V(r) \rightarrow \boxed{V(z) \cong \frac{1}{4\pi\varepsilon_0}\frac{1}{z^3}\frac{k\pi^2 R^5}{48}}$$