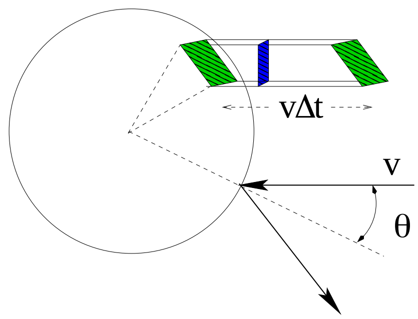
Problem 10-14 solution

Number of particles in the volume $\Delta V = v\Delta t\Delta A\cos\theta$ is $(n\Delta V) \Rightarrow$ the number of particles incident of the patch of spherical surface ΔA is

$$n\Delta V \;=\; nv\Delta A\cos\theta\Delta t$$

$$\Delta V = v\Delta t\,\Delta A = v\Delta t\,\Delta A\cos\theta = v\Delta t\, r^2\Delta\Omega\,\cos\theta$$



The change of z-component of momentum of particle incident at angle θ is $\Delta p_z = mv(1+\cos 2\theta)$ so the total change of momentum of particles incident at ΔA is

$$\Delta p_z \;=\; nmv^2\Delta t\Delta A\cos\theta(1+\cos 2\theta)$$

Since $\Delta A = r^2\sin\theta\Delta\theta\Delta\phi$ the change of momentum of all particles colliding with the satellite is

$$\int_0^{\pi/2}\sin\theta d\theta\int_0^{2\pi}d\phi\; nmv^2\Delta t r^2(1+\cos 2\theta)\cos\theta \;=\; \Delta t\; 2\pi mnv^2\int_0^{\pi/2}\sin\theta\cos\theta(1+\cos 2\theta)d\theta \tag{1}$$

an therefore the force is

$$F_z \;=\; \frac{\Delta p_z}{\Delta t} \;=\; 2\pi mnv^2\int_0^{\pi/2}\sin\theta\cos\theta(1+\cos 2\theta)d\theta \;=\; \pi mnv^2$$

Check for number of particles colliding with the satellite in a time Δt: integral (1) without $\Delta p_z = mv(1+\cos 2\theta)$

$$nv\Delta t r^2\int_0^{\pi/2}\sin\theta\cos\theta d\theta\int_0^{2\pi}d\phi \;=\; n\pi r^2\Delta t$$

which is cross section πr^2 times length $v\Delta t$ (without $\cos\theta$ one would get two times more).

Problem 10-21 solution

The problem is very similar to that of Sect. 10-6. The flow of the number of particles is given by Eq. (10-28)

$$\Gamma \;=\; -\frac{1}{3}\bar{v}l\frac{dn}{dy}$$

so the flow of the mass is

$$m\Gamma \;=\; \frac{m}{3}\bar{v}l\frac{dn}{dy}$$

We need to re-express the equation for in terms of P rather than n. Since $P = nkT$ we get $\frac{dn}{dy} = \frac{1}{kT}\frac{dP}{dy}$ and therefore

$$m\Gamma \;=\; \frac{m\bar{v}l}{3kT}\frac{dP}{dy}$$

so the coefficient is $\frac{m\bar{v}l}{3kT}$.