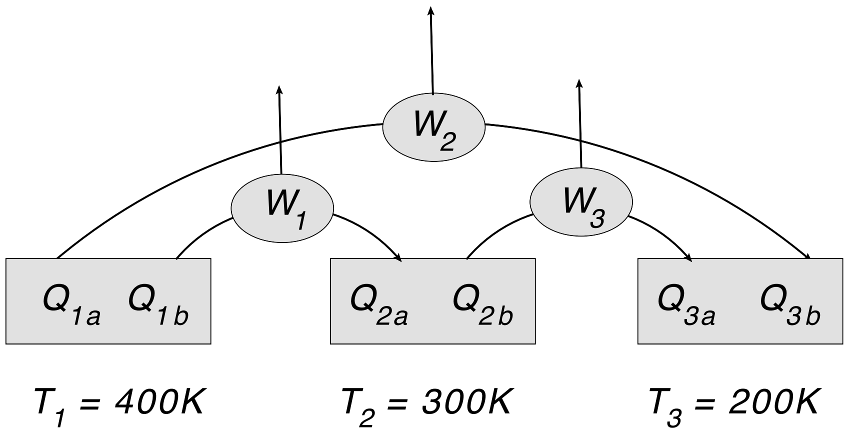
Problem 5-7 solution



Solution #1 (a)
We assume that the engines are Carnot engines, then

$$Q_{2a} = \frac{T_2}{T_1}Q_{1b} = \frac{3Q_{1b}}{4}, \quad Q_{3b} = \frac{T_3}{T_1}Q_{1a} = \frac{Q_{1a}}{2}, \quad Q_{3a} = \frac{T_3}{T_2}Q_{2b} = \frac{2Q_{2b}}{3}$$
$$\Rightarrow W_1 = Q_{1b} - Q_{2a} = \frac{Q_{1b}}{4}, \quad W_2 = Q_{1a} - Q_{3b} = \frac{Q_{1a}}{2}, \quad W_3 = Q_{2b} - Q_{3a} = \frac{Q_{2b}}{3}$$

Also,

$$Q = Q_{1a} + Q_{1b} = 1200J$$
$$W = W_1 + W_2 + W_3 = \frac{Q_{1a}}{2} + \frac{Q_{1b}}{4} + \frac{Q_{2b}}{3} = 200J$$

We get

$$W = \frac{Q_{1a}}{2} + \frac{Q_{1b}}{4} + \frac{Q_{2b}}{3} = \frac{Q_{1a} + Q_{1b}}{2} - \frac{Q_{1b}}{4} + \frac{Q_{2b}}{3} = \frac{Q}{2} - \frac{Q_{2a}}{3} + \frac{Q_{2b}}{3}$$
$$\Rightarrow \quad Q_{2a} - Q_{2b} = 3\left(\frac{Q}{2} - W\right) = 1200J$$

$\Rightarrow$ 1200J is rejected at the second reservoir.
Next, from the overall conservation of energy

$$W = Q_{1a} + Q_{1b} - Q_{2a} + Q_{2b} - Q_{3a} - Q_{3b} = -Q_{3a} - Q_{3b} = 200J \Rightarrow (Q_{3a} + Q_{3b}) = -200J$$

$\Rightarrow$ 200J of heat is absorbed at the third reservoir

(b), (c)

$$\Delta S_1 = \frac{Q_{1a} + Q_{1b}}{T_1} = 3\frac{J}{K}, \quad \Delta S_2 = \frac{Q_{2a} - Q_{2b}}{T_2} = -4\frac{J}{K}, \quad \Delta S_3 = \frac{Q_{3a} + Q_{3b}}{T_3} = 1\frac{J}{K}$$

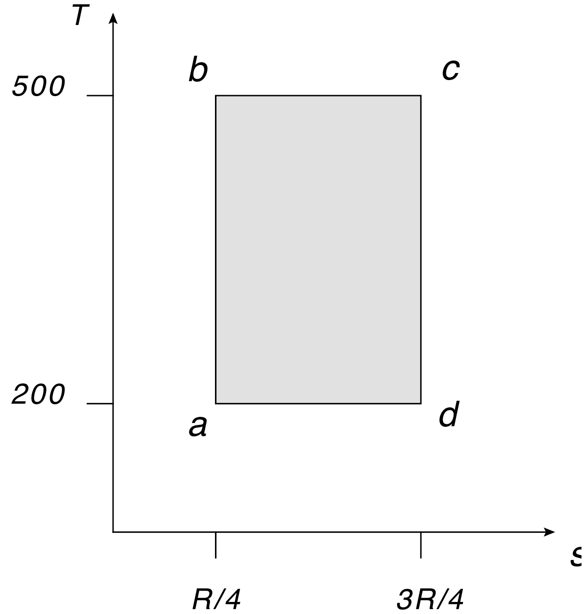
$\Rightarrow \Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = 0$ as it should be for a reversible process.

Solution #2
Let us denote heats absorbed by the engine as Q_1, Q_2, and Q_3 (so $Q_1 = Q_{1a} + Q_{1b}$, $Q_2 = -Q_{2a} + Q_{2b}$, $Q_3 = -Q_{3a} - Q_{3b}$). If the process is reversible, change of entropy is zero so we get

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = 0 \quad \text{and} \quad Q_1 + Q_2 + Q_3 = W$$
$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_1 + Q_2}{T_3} + \frac{W}{T_3} = 0 \Rightarrow 3 + \frac{Q_2}{300} - 6 + 1 = 0 \Rightarrow Q_2 = -1200, \; Q_3 = 200$$

so 1200J is rejected at reservoir 2 and 200J is absorbed at reservoir 3.

Problem 5-13 solution



(a): heat engine

(b)

$$
\begin{aligned}
Q_{a\to b} &= 0, \\
Q_{b\to c} &= T_2(s_2 - s_1) \;=\; 250R \;\simeq\; 2.1\times 10^6 J, \\
Q_{c\to d} &= 0, \\
Q_{d\to a} &= T_1(s_1 - s_2) \;=\; -100R \;\simeq\; -8.3\times 10^5 J
\end{aligned}
$$

(c)

$$\eta \;=\; \frac{W}{Q_{b\to c}} \;=\; \frac{Q_{b\to c} + Q_{d\to a}}{Q_{b\to c}} \;=\; \frac{T_2 - T_1}{T_2} \;=\; 60\%$$

(d)
In reverse

$$c \;=\; \frac{Q_{a\to d}}{Q_{b\to c} - Q_{a\to d}} \;=\; \frac{T_2}{T_2 - T_1} \;=\; \frac{2}{3} \;\simeq\; 67\%$$