A car (of mass m) is moving along a totally straight but "hilly" country road. We can describe the position of the car by its projection x onto the x-axis and the height $h(x)$ of the road above sea level. The car starts out with zero velocity at some height H at $x = 0$ and then rolls without any forces other than gravity and the normal force acting on it (*i.e.*, the engine is off and there is no friction).

1. Using x as the generalized coordinate, write down the Lagrangian $\mathcal{L}$. Solve the Euler-Lagrange equation for $\ddot{x}$. Express your result in terms of the incline angle θ that the road makes with the horizontal.

2. From the same Lagrangian, determine also the energy

$$E = \dot{x}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \mathcal{L} \, ,$$

and solve for the square of the generalized velocity, $\dot{x}^2$. (Why is energy conserved in this case ?)