Phys. 807 — Statistical Mechanics

Solution.

Newton's 2nd law:

$$m\frac{v^2}{R} \;=\; \frac{GMm}{R^2} \quad \Rightarrow \quad v^2 \;=\; \frac{GM}{R}$$

Energy

$$E \;=\; \frac{mv^2}{2} - \frac{GMm}{R} \;=\; -\frac{GMm}{2R}$$

After one revolution

$$E(t+T) - E(t) \;\simeq\; -Av^{\alpha}(t)2\pi R(t)$$

Equation

$$Av^{\alpha}(t)2\pi R(t) \;=\; -T\frac{d}{dt}E(t) \;=\; T\frac{d}{dt}\frac{GMm}{2R(t)} \;=\; -T\frac{GMm}{2R^2(t)}\frac{dR}{dt} \;=\; CT\frac{GMm}{2R^2(t)} \;=\; \frac{\pi C}{v(t)}\frac{GMm}{R(t)}$$

$$\Rightarrow\; v^{\alpha+1}(t)R^2(t) \;=\; v^{\alpha+1}\frac{G^2M^2}{v^4} \;=\; GMm\frac{C}{2A} \quad \Rightarrow \quad 2v^{\alpha-3}\frac{GMA}{mC} \;=\; 1$$

and therefore

$$\alpha \;=\; 3, \qquad A \;=\; \frac{mC}{2GM}$$