Phys. 807 — Statistical Mechanics

Solution.

The equation of motion is (2.44) from the lecture notes

$$m\ddot{\vec{r}} = m\vec{g}+\vec{F}_c+\vec{N} = 2mv\omega_e\cos\theta(\hat{e}_x\sin\phi-\hat{e}_y\cos\phi)+\hat{e}_z(-mg+N+2mv\omega_e\sin\theta\sin\phi)$$

Since the puck is not jumping, the normal force equilibrates $mg-2mv\omega_e\sin\theta\sin\phi$ and the motion occurs in the xy plane. The corresponding equation is

$$m\hat{e}_x\ddot{x}+m\hat{e}_y\ddot{y} = 2mv\omega_e\cos\theta(\hat{e}_x\sin\phi-\hat{e}_y\cos\phi)$$

In components

$$\begin{aligned}\ddot{x} &= 2v\omega_e\cos\theta\sin\phi = 2\omega_e\cos\theta\dot{y}\\ \ddot{y} &= 2v\omega_e\cos\theta\cos\phi = -2\omega_e\cos\theta\dot{x}\end{aligned}$$

so we get the equations

$$\begin{aligned}\dddot{x} &= -(2\omega_e\cos\theta)^2\dot{x}, \qquad \dddot{y} = -(2\omega_e\cos\theta)^2\dot{y}\\ \Rightarrow \dot{x} &= v_0\cos(2\omega_e t\cos\theta+\delta), \qquad \dot{y} = -v_0\sin(2\omega_e t\cos\theta+\delta)\end{aligned}$$

where v_0 is the initial velocity. The solutions are

$$\begin{aligned}x(t) &= x_0+\frac{v_0}{2\omega_e\cos\theta}\sin(2\omega_e t\cos\theta+\delta),\\ y(t) &= y_0+\frac{v_0}{2\omega_e\cos\theta}\cos(2\omega_e t\cos\theta+\delta)\end{aligned}$$

This is the circular motion around the point (x_0, y_0) with radius $R = \frac{v_0}{2\omega_e\cos\theta}$ and frequency $\omega = 2\omega_e\cos\theta$.