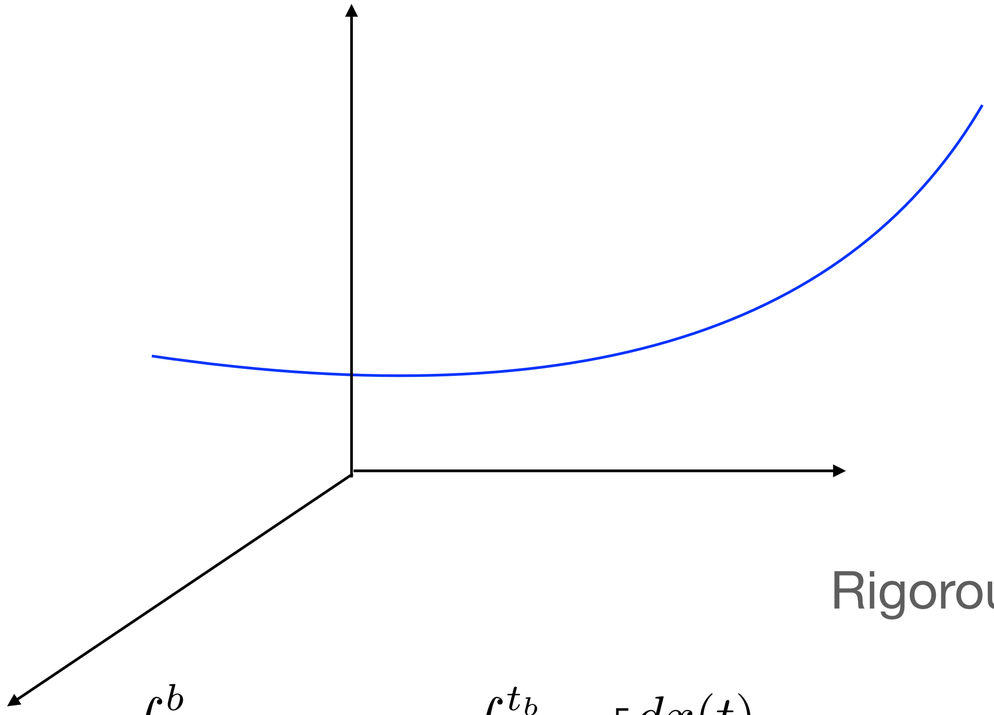
Line integrals



$$x = x(t), \quad y = y(t), \quad z = z(t), \qquad t \equiv \text{parameter}$$

Rigorous definition of line integral:

$$\int_a^b \vec{dl}\cdot\vec{f}(\vec{r}) \equiv \int_{t_a}^{t_b} dt \left[\frac{dx(t)}{dt} f_1\big(x(t),y(t),z(t)\big) + \frac{dy(t)}{dt} f_2\big(x(t),y(t),z(t)\big) + \frac{dz(t)}{dt} f_3\big(x(t),y(t),z(t)\big)\right]$$

The integral does not depend on the choice of parameter:

$$\int_{t'_a}^{t'_b} dt' \left[\frac{dx(t')}{dt'} f_1\big(x(t'),y(t'),z(t')\big) + \ldots\right] = \text{change of variables } t' = t'(t) =$$

$$= \int_{t_a}^{t_b} dt \, \frac{dt'(t)}{dt} \left[\frac{dx(t'(t))}{dt'} f_1\big(x(t'(t)),y(t'(t)),z(t'(t))\big) + \ldots\right] \overset{\text{chain rule}}{=} \int_{t_a}^{t_b} dt \left[\frac{dx(t)}{dt} f_1\big(x(t),y(t),z(t)\big) + \ldots\right]$$