Problem 2 UIN _____

Consider a beam of electrons of density n (the number of electrons per unit volume), velocity $\mathbf{v}$, and cross-sectional area S.

(1) Calculate the current I and charge density ρ in the laboratory frame K, and the charge density ρ_0 in the rest frame K_0 of the electrons.

(2) A positron is moving at a distance d from the beam with the velocity $-\mathbf{v}$ in K (i.e., the positron velocity is equal in magnitude to that of the electrons, but in the opposite direction). Calculate the force acting on the positron in K by first calculating it in the rest frame K' of the positron and then transforming it back to K.

(3) Assuming $d > S^{1/2}/\pi$, calculate the electric and magnetic fields generated by the beam of electrons in K, and obtain the force acting on the positron in this frame. Does it agree with the force obtained in 2. above?

HINT: You may want to use the Lorentz transformation for the electric and magnetic fields:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}, \qquad \mathbf{E}'_{\perp} = \gamma(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{H}/c),$$

$$\mathbf{H}'_{\parallel} = \mathbf{H}_{\parallel}, \qquad \mathbf{H}'_{\perp} = \gamma(\mathbf{H}_{\perp} - \mathbf{v} \times \mathbf{E}/c),$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$.