Problem 2 UIN _____

An approximately monochromatic plane wave packet in one dimension has the instantaneous form

$$u(x,0) = f(x)e^{ik_0 x}$$

with the modulation envelope $f(x)$. For

$$f(x) = \begin{cases} N(1-\alpha^2 x^2) & \alpha^2 x^2 \leqslant 1 \ , \\ 0 & \alpha^2 x^2 \geqslant 1 \ . \end{cases}$$

a) calculate the wave-number spectrum $|A(k)|^2$ of the packet,

b) sketch $|u(x,0)|^2$ and $|A(k)|^2$,

c) using the expressions for $|u(x,0)|^2$ and $|A(k)|^2$), evaluate explicitly the rms deviations Δx and Δk from their mean values and test the inequality

$$\Delta x \Delta k \geq 1/2 \ .$$