HW 11. Due Mon April 12 by 5 p.m. by e-mail

Problem.

A 500 m long spaceship is moving with 90% of the speed of light relative to Earth. The two lasers send simultaneous (according to the space ship) flashes of light back to Earth - one from the front tip of the spaceship and one from the rear end. After some time, these signals are received by observatory on Earth. What will be the time difference between receiving these two signals on Earth?

Solution # 1.

Event #1: x_1, t_1. Event #2: x_2, t_2

$$x_1 = \gamma(x_1' + vt'), \quad t_1 = \gamma(t' + \frac{v}{c^2}x_1), \qquad x_2 = \gamma(x_2' + vt'), \quad t_2 = \gamma(t' + \frac{v}{c^2}x_2)$$

Rime difference between receiving these two signals on Earth is

$$t_2 + \frac{x_2}{c} - t_1 - \frac{x_1}{c} \;=\; \frac{x_{21}}{c} + t_{21} \;=\; \frac{x_{21}}{c}(1 + \frac{v}{c}) \;=\; \gamma\frac{x_{21}'}{c}(1 + \frac{v}{c})$$

The number is

$$\frac{5}{3}10^{-6}\frac{19}{\sqrt{19}} \;=\; 7.26 \times 10^{-6}s$$

Solution # 2: in spaceship's frame.

Let the first signal reach moving Earth at time τ_1'. The second signal will catch with Earth at time $\tau_2' = \tau_1' + \frac{v\tau_{21}' + x_{21}'}{c}$

$$\Rightarrow \; \tau_{21}' \;=\; \frac{x_{21}'}{c - v} \quad \Rightarrow \quad \tau_{21} \;=\; \frac{\tau_{21}'}{\gamma} \;=\; \frac{x_{21}'}{c}\frac{1}{\gamma(1 - \frac{v}{c})} \;=\; \text{same result}$$