Phys. 804 — Classical Electrodynamics

HW 5 Solution

$$A(k) \;=\; \int_{\frac{-1}{a}}^{\frac{1}{a}} dx\; e^{i(k_0-k)x}(1-a^2x^2) \;=\; 2\int_0^{\frac{1}{a}} dx\;\; \cos qx(1-a^2x^2) \;=\; \frac{4a}{q^3}(\,-q\cos\frac{q}{a}+a\sin\frac{q}{a}), \qquad q\equiv|k-k_0|$$

$$\langle x\rangle \;=\; 0 \;\;\Rightarrow\;\; \Delta x^2 \;=\; \frac{\int_{\frac{-1}{a}}^{\frac{1}{a}} dx\; x^2(1-a^2x^2)^2}{\int_{\frac{-1}{a}}^{\frac{1}{a}} dx\;(1-a^2x^2)^2} \;=\; \frac{(16/105a^3)}{(16/15a)} \;=\; \frac{1}{7a^2}$$

$$\langle k\rangle \;=\; \frac{\int_{-\infty}^{\infty} dk\; k\left[\frac{4a}{q^3}(\,-q\cos\frac{q}{a}+a\sin\frac{q}{a})\right]^2}{\int_{-\infty}^{\infty} dk\;\left[\frac{4a}{q^3}(\,-k\cos\frac{q}{a}+a\sin\frac{q}{a})\right]^2} \;=\; \frac{\int_{-\infty}^{\infty} dk\;(k+k_0)\left[\frac{4a}{k^3}(\,-q\cos\frac{k}{a}+a\sin\frac{|k|}{a})\right]^2}{\int_{-\infty}^{\infty} dk\;\left[\frac{4a}{k^3}(\,-k\cos\frac{k}{a}+a\sin\frac{|k|}{a})\right]^2} \;=\; k_0$$

$$\Delta k^2 \;=\; \frac{\int_{-\infty}^{\infty} dk\;(k-k_0)^2\left[\frac{4a}{q^3}(\,-q\cos\frac{q}{a}+a\sin\frac{q}{a})\right]^2}{\int_{-\infty}^{\infty} dk\;\left[\frac{4a}{q^3}(\,-k\cos\frac{q}{a}+a\sin\frac{q}{a})\right]^2} \;=\; \frac{\int_{-\infty}^{\infty} dk\; k^2\left[\frac{4a}{k^3}(\,-q\cos\frac{k}{a}+a\sin\frac{|k|}{a})\right]^2}{\int_{-\infty}^{\infty} dk\;\left[\frac{4a}{k^3}(\,-k\cos\frac{k}{a}+a\sin\frac{k}{a})\right]^2}$$

$$=\; \frac{\int_0^{\infty} dk\; k^2\left[\frac{4a}{k^3}(\,-q\cos\frac{k}{a}+a\sin\frac{k}{a})\right]^2}{\int_0^{\infty} dk\;\left[\frac{4a}{k^3}(\,-k\cos\frac{k}{a}+a\sin\frac{k}{a})\right]^2} \;=\; \frac{8\pi a/3}{16\pi/(15a)} \;=\; \frac{5a^2}{2}$$

$$\Rightarrow\;\; \Delta x\Delta k \;=\; \sqrt{\frac{5}{14}} \;>\; \frac{1}{2}$$