Problem. A beam of spineless nuclear particles of mass m and momentum p is directed along the z-axis. The particles collide with an aligned diatomic molecule but interact only with the nuclei of the molecule. If the nuclei are taken to be at y = b and y = ?b, and the constant a is positive, the interaction potential is given by

$$V(\vec{r}) \;=\; a\delta(y-b)\delta(x)\delta(z) + a\delta(y+b)\delta(x)\delta(z)$$

Calculate the scattering amplitude and the differential cross section in the Born approximation.

Solution.

In the Born approximation

$$f_k(\theta,\phi) \;=\; -\frac{m}{2\pi\hbar^2}V(\vec{k}-\vec{k}') \;=\; -\frac{m}{2\pi\hbar^2}\int d^3r'\; e^{i\vec{r}'\cdot(\vec{k}-\vec{k}')}V(\vec{r}')$$

and therefore

$$\begin{aligned}
f_k(\theta,\phi) \;&=\; -\frac{m}{2\pi\hbar^2}\int d^3r'\; e^{i\vec{r}'\cdot(\vec{k}-\vec{k}')}[a\delta(y'-b)\delta(x')\delta(z') - a\delta(y'+b)\delta(x')\delta(z')] \\
&=\; -\frac{m}{2\pi\hbar^2}\int dx'dy'dz'\; e^{-ix'k'_x-iy'k'_y+iz(k-k'_z)}[a\delta(y'-b)\delta(x')\delta(z') + a\delta(y'+b)\delta(x')\delta(z')] \\
&=\; -\frac{ma}{2\pi\hbar^2}(e^{-ibk'_y}+e^{ibk'_y}) \;=\; i\frac{ma}{\pi\hbar^2}\cos bk'_y \;=\; \frac{ima}{\pi\hbar^2}\sin(bk\sin\theta\sin\phi)
\end{aligned}$$

so the differential cross section is

$$\frac{d\sigma}{d\Omega} \;=\; |f_k(\theta,\phi)|^2 \;=\; \frac{m^2a^2}{\pi^2\hbar^4}\cos^2(bk\sin\theta\sin\phi)$$