Solution to HW 3

The differential cross section for a general $2 \Rightarrow 2$ particle cross section is calculated in Appendix A. For our case

$$\left(\frac{d\sigma}{d\Omega}\right)_{"1"} = \frac{1}{64\pi^2 s}|T|^2 \tag{1}$$

where $s = (E_1 + E_1')^2$ and the label "1" means that we catch the first particle (when it flies into the spherical angle $d\Omega$). The relevant diagram is shown in Fig.(1). There are no diagrams of the Fig. $24b$ type since the particles are not identical and the diagram of the Fig. $24c$ type is absent because I did not specify that particle "1" can emit π-meson and convert into particle "2". The transition matrix is just the amputated modified Green function ,

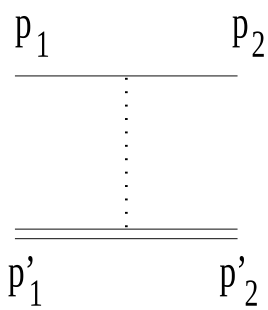

FIG. 1. Feynman diagram for the $1, 2 \Rightarrow 1, 2$ scattering. Particle "1" is denoted by a single line and particle "2" by a double one.

$$T(p_2, p_2'; p_1, p_1') = \frac{\lambda^2}{m^2 - t - i\epsilon} \tag{2}$$

where $t = -4|\vec{p}_1|^2 \sin^2 \frac{\theta}{2}$, same as before. Thus,

$$\left(\frac{d\sigma}{d\Omega}\right)_{"1"} = \frac{\lambda^4}{64\pi^2 s}\left(\frac{1}{m^2 + 4|\vec{p}_1|^2 \sin^2 \frac{\theta}{2}}\right)^2 \tag{3}$$

If our detector cannot distringuish between the particles "1" and"2" you must add the corresponding cross section for the scattering into $\pi - \theta$ angle (since in this case the "2" particle will get into our detector located at angle θ):

$$\left(\frac{d\sigma}{d\Omega}\right)_{1 \text{ or } 2} = \frac{\lambda^4}{64\pi^2 s}\left[\left(\frac{1}{m^2+4|p_1|^2\sin^2\frac{\theta}{2}}\right)^2 + \left(\frac{1}{m^2+4|p_1|^2\cos^2\frac{\theta}{2}}\right)^2\right] \tag{4}$$

Note that unlike the contributions of the Fig. 24a and b we add the cross sections rather than the amplitudes since our particles are, in principle, distinguishable and it is only because we decided to save on a cheap detector we cannot separate them.

The total cross section is just

$$\sigma_{\text{tot}} = \int d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{\text{"1"}} = \frac{\lambda^4}{32\pi s}\int_0^\pi d\theta \sin\theta \left(\frac{1}{m^2+2|\vec{p}_1|^2(1-\cos\theta)}\right)^2 \tag{5}$$

Performing the integration, we get

$$\sigma_{\text{tot}} = \frac{\lambda^4}{16\pi s}\frac{1}{m^2(m^2+4|\vec{p}_1|^2)} \tag{6}$$