Solution to HW 3

Find the differential cross section for the $\pi^+\pi^- \Rightarrow \pi^+\pi^-$ scattering in the first nontrivial order in perturbation theory.

The differential cross section for a general $2 \Rightarrow 2$ particle cross section is calculated in Appendix A. For our case

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{64\pi^2 s}|T|^2 \tag{1}$$

where $s = (E_1 + E_1')^2$. The relevant diagrams are shown in Fig. 1 below. The transition

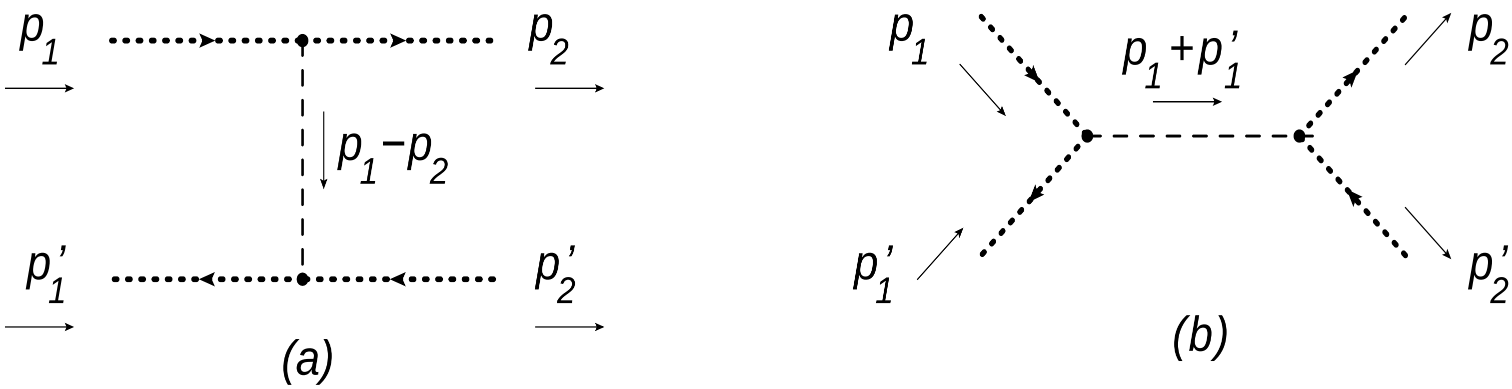

FIG. 1. Feynman diagrams for $\pi^+\pi^- \to \pi^+\pi^-$ scattering.

matrix is the amputated reduced Green function:

$$\begin{aligned} T(p_2, p_2'; p_1, p_1') &= \frac{e^2}{(p_1 - p_2)^2 + i\epsilon}(p_1 + p_2)\cdot(-p_1' - p_2') + \frac{e^2}{(p_1 + p_1') + i\epsilon}(p_1 - p_1')\cdot(p_2 - p_2') \\ &= -\frac{e^2}{t}(p_1 + p_2)\cdot(p_1' + p_2') + \frac{e^2}{s}(p_1 - p_1')\cdot(p_2 - p_2'). \end{aligned} \tag{2}$$

Using $p_1 \cdot p_1' = p_2 \cdot p_2' = \frac{s}{2} - m^2$, $p_1 \cdot p_2 = p_1' \cdot p_2' = m^2 - \frac{t}{2}$ and $p_1 \cdot p_2' = p_2 \cdot p_1' = m^2 - \frac{u}{2}$ we get

$$T(p_2, p_2'; p_1, p_1') \;\; = \;\; -\, e^2\Big[\frac{s-u}{t} + \frac{t-u}{s}\Big] \tag{3}$$

Note that the expressions for diagrams Fig. 1a and Fig1b are related by crossing symmetry $p_2 \leftrightarrow -p_1' \;\Leftrightarrow\; s \leftrightarrow t$.

Using Eq. (4.121) from the lecture notes

$$s = 4E_1^2, \quad t = -4|\vec{p}_1|^2 \sin^2\left(\frac{\theta}{2}\right), \quad u = -4|\vec{p}_1|^2 \cos^2\left(\frac{\theta}{2}\right) \tag{4}$$

we get

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{e^4}{64\pi^2 s}\left[\frac{1+\cos^2\frac{\theta}{2} - \frac{m^2}{|\vec{p}|^2}}{\sin^2\frac{\theta}{2}} - \frac{4|\vec{p}|^2}{s}\cos\theta\right] \tag{5}$$