Solution to HW 6

The bispinor for an electron moving in Y direction with rest-frame spin in X direction can be obtained from Eq. (8.53):

$$u(p,s) = \frac{1}{\sqrt{2(p_0+m)}}\begin{pmatrix}(m+p_\mu\sigma^\mu\bar\sigma_0)\kappa\\(m+p_\mu\bar\sigma^\mu\sigma_0)\kappa\end{pmatrix} = \frac{1}{\sqrt{2(p_0+m)}}\begin{pmatrix}(m+p_0-\vec p\cdot\vec\sigma)\begin{pmatrix}\cos(\theta/2)\\e^{i\phi}\sin(\theta/2)\end{pmatrix}\\(m+p_0+\vec p\cdot\vec\sigma)\begin{pmatrix}\cos(\theta/2)\\e^{i\phi}\sin(\theta/2)\end{pmatrix}\end{pmatrix} \tag{1}$$

where $\theta=\frac{\pi}{2}$, $\phi=0$, and $p=(p_0,0,p,0)$. In explicit form $\kappa=\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$ and therefore

$$u(p,s) = \ = \frac{1}{2\sqrt{p_0+m}}\begin{pmatrix}(m+p_0-\vec p\cdot\vec\sigma)\begin{pmatrix}1\\1\end{pmatrix}\\(m+p_0+\vec p\cdot\vec\sigma)\begin{pmatrix}1\\1\end{pmatrix}\end{pmatrix} = \frac{1}{2\sqrt{(p_0+m)}}\begin{pmatrix}m+p_0+ip\\m+p_0-ip\\m+p_0-ip\\m+p_0+ip\end{pmatrix} \tag{2}$$

so the wavefunction is

$$\psi^e(x) = \frac{1}{2\sqrt{(p_0+m)}}\begin{pmatrix}m+p_0+ip\\m+p_0-ip\\m+p_0-ip\\m+p_0+ip\end{pmatrix}\frac{1}{\sqrt{2p_0}}e^{-i(p_0t-py)} \tag{3}$$

The bispinor for a positron moving in X direction with rest-frame spin in Y direction can be obtained from Eqs. (8.56) and (8.58):

$$\begin{aligned}\bar v(p,s) &= \frac{1}{\sqrt{2(p_0+m)}}[\kappa^\dagger(m\sigma_0+p_\mu\bar\sigma^\mu),\kappa^\dagger(-m-\bar\sigma_0p_\mu\sigma^\mu) \ = \\ &\frac{1}{\sqrt{2(p_0+m)}}\big[\big(-e^{i\phi}\sin\frac{\theta}{2},\cos\frac{\theta}{2}\big)(m+p_0+\vec p\cdot\vec\sigma);\big(-e^{i\phi}\sin\frac{\theta}{2},\cos\frac{\theta}{2}\big)(-m-p_0+\vec p\cdot\vec\sigma)\big]\end{aligned} \tag{4}$$

where now $\theta=\frac{\pi}{2}$, $\phi=\frac{\pi}{2}$, and $p=(p_0,p,0,0)$. In explicit form $\kappa^\dagger=\frac{1}{\sqrt2}(-i,1)$ (recall that $\kappa^\dagger\sigma_y=-\kappa^\dagger$) and therefore

$$\begin{aligned}\bar v(p,s) &= \frac{1}{2\sqrt{p_0+m}}\big[(-i,1)(m+p_0+p\sigma_x);(-i,1)(-m-p_0+p\sigma_x))\big]\\ &= \frac{1}{2\sqrt{p_0+m}}\big[-i(m+p_0)+p,\ m+p_0-ip_1,\ (m+p_0)+p,\ -(m+p_0)-ip\big]\end{aligned} \tag{5}$$

so the positron wavefuction takes the form:

$$\underline{\psi}^p(x) = \frac{1}{2\sqrt{p_0+m}}\big[-i(m+p_0)+p,\ m+p_0-ip_1,\ i(m+p_0)+p,\ -(m+p_0)-ip\big]\frac{e^{-i(p_0t-px)}}{\sqrt{2p_0}} \tag{6}$$